

Article

A Practical Research on the Development of a Model for High School-University Mathematics Transition Course for Inclusive Education

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Abstract: The smooth transition from high school mathematics to university mathematics should not solely rely on remedial teaching in the initial stage of university. High school mathematics teachers can effectively promote a better connection between high school and university mathematics by leveraging high school mathematics. This article explores a universal embedded model for the connection between high school and university mathematics. Based on real problems in high school mathematics teaching, it collaborates with university mathematics teachers to develop high school-university connection lessons. Taking the outer product of vectors as an example, it goes through the stages of preliminary preparation, development design, teaching implementation, and evaluation feedback, and constructs a general path for high school mathematics teachers to develop high school-university connection lessons. This provides a research foundation for integrating higher mathematics thinking into high school mathematics teaching.

Keywords: High school mathematics; University mathematics; Example of bridging courses; Vector cross product

1. Introduction

The issue of the transition between high school mathematics and university mathematics has been widely discussed by mathematicians and mathematics educators for a long time [1-2]. Researchers have analyzed the transition between high school and university mathematics from different perspectives, but there is still no universally accepted analytical framework at present [3]. The practice of the transition between high school and university mathematics exhibits various manifestations. Based on the academic stage and purpose where the transition activities occur, it can be summarized into two parts: one is the transition courses from high school mathematics to university mathematics offered by universities; the other is the support provided by high school for students to access and learn university mathematics. The former mainly refers to remedial and transitional measures provided by universities during university studies, while the latter is more inclined to provide enrichment and accelerated transition measures for high school students. Transitioning from high school mathematics to university mathematics has always been a hot topic in international mathematics education research [4-5]. The research has expanded from mainly focusing on the epistemological and cognitive difficulties faced by university freshmen to social and cultural aspects and emotions. Compared with this, the transitional measures provided by high school for students to access and learn university mathematics are more often related to mathematics gifted education and the cultivation of outstanding and innovative talents. For instance, successful high school-university cooperative programs, such as the "Fudan-SHS Mentorship Program" and the pre-university mathematics courses in Qingdao Academy, have demonstrated significant value in bridging the gap for capable students [6,7]. For

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example, the transition methods such as university mathematics pre-requisite courses and mathematics competition training offered in high school are usually targeted at a small number of mathematically talented students with sufficient abilities, and the range of beneficiaries is relatively narrow [8].

At present, China's higher education has entered the massification stage. One of the goals of high school education is to prepare students for university studies and careers. Specifically in the field of mathematics, it is appropriate for high school mathematics teaching to moderately incorporate advanced mathematical knowledge or thinking, which is beneficial for many capable high school students to enter universities and adapt to the study of advanced mathematics. The research team of "Key Mathematical Concepts in the Transition from High School to University" has shown that, to help university freshmen achieve a smooth transition in university mathematics learning, most university teachers believe that changes should be made at the high school level, while a few suggest strengthening communication with high school teachers [9]. Currently, high school mathematics elective courses include five categories, which can be regarded as prerequisites for university mathematics courses and are suitable for providing course content for students interested in different majors. The "General High School Mathematics Curriculum Annotation (2017 Edition, Revised in 2020)" (hereinafter referred to as "Curriculum Annotation 2017 Edition 2020 Revised") defines the positioning of elective courses as: "To provide guidance for students' future development, to offer a platform for students to showcase their mathematical abilities, to provide options for students to develop their interest in mathematics, and to serve as a reference for university independent recruitment". Given the significant functions of elective courses, in reality they are often poorly implemented because of factors such as the absence of a mathematics examination in the college entrance examination, high academic pressure, limited teaching time, and insufficient teacher reserves [10].

To address the practical difficulties faced by high school mathematics teachers in integrating advanced mathematics knowledge into their teaching, a cooperative exchange between high school and university mathematics teachers to establish a teaching and research community is an effective measure [11]. Relevant textbooks and monographs on the transition between high school and university mathematics also provide valuable reference materials for such collaborative efforts [12-14]. Mu Chunxia et al. conducted a case study on the cooperation between high school and university mathematics teachers, and found that in the cooperative community, the role of teachers changed from passive to active, taking on the roles of "problem proposers" and "solvers of cooperative problems". Researchers can better understand the teaching reality and conduct targeted research [15]. In China, there is no shortage of precedents for cooperation between high schools and universities, and many successful experiences can be drawn upon [16]. For example, Professor Wang Xiaokun from East China Normal University's HPM lesson case development team gathered the wisdom of front-line teachers, research coordinators, and university teachers, and through long-term practice, formed a lesson case development process of topic selection and preparation, discussion and design, implementation and feedback, and summary and writing [17].

High school mathematics teachers can effectively facilitate a better connection between high school and university mathematics by reasonably arranging teaching content and innovating teaching methods [18]. Based on the actual situation of most high school mathematics teaching, in order to provide more students with the opportunity to access university mathematics knowledge and enhance their willingness to pursue further studies, an exploratory approach is sought to develop a feasible model for the connection between high school and university mathematics. This model relies on the content of high school mathematics courses and uses real problems in high school teaching as the starting point and opportunity to develop high school-university connection lesson examples. These types of lesson examples usually can be completed within 2 to 3 class hours. High school mathematics teaching can be flexibly embedded into regular teaching as needed, featuring micro-topic courses, and using "the outer product of vectors" as an example, the

complete process of university mathematics teachers and high school mathematics teachers collaborating to develop inclusive embedded high school-university connection lesson examples is demonstrated, in order to address specific research questions: How do high school and university mathematics teachers collaborate to develop inclusive embedded high school-university connection lesson examples? What is the appearance of high school-university mathematics connection lesson examples? What is the effect of implementing high school-university mathematics connection lesson examples?

2. Research Design

2.1. Selection of Practical Teachers

Teacher D, a practicing teacher with 14 years of high school teaching experience, is a district-level key teacher and a part-time educational researcher. With solid subject expertise and rich teaching experience, Teacher D is currently pursuing a doctoral degree in education at a certain university and also teaches high school mathematics. She has accumulated certain experience in transitional teaching between high school and university mathematics. The high school where she teaches has a history of over 50 years and possesses a rich cultural heritage. It has developed its own unique educational characteristics and was once recognized by the Ministry of Railways as one of the first key high schools. After being evaluated and inspected by the provincial education department, it was promoted to a first-class comprehensive high school in the province. The quality of its student body is at the upper-middle level in the province.

2.2. Real Problem-Driven Linkage

In Chapter 1, Section 1.4.2 of *Selective Compulsory Mathematics for Senior High School (Edition A)*, published by People's Education Press in 2019, the topic discussed is the study of angle problems using spatial vectors. When explaining the problem of plane angles, Teacher D took into account that the dihedral angle is defined by the angle formed by two half-planes, and calculating the size of the angle between two planes is also a frequently tested topic in the college entrance examination mathematics. Given that the textbook did not provide corresponding examples, Teacher D decided to adapt the material to local conditions and designed the calculation of the cosine value of the angle between two planes based on the problem statement of Example 8 in the textbook. The adapted question is as follows:

As shown in Figure 1, in the right triangular prism $ABC-A_1B_1C_1$, where $AC=CB=2$, $AA_1=3$, and $\angle ACB = 90^\circ$, Point P is the midpoint of BC . Points Q and R are located on edges AA_1 and BB_1 respectively. A_1Q is twice the length of AQ , and BR is twice the length of R_1B .

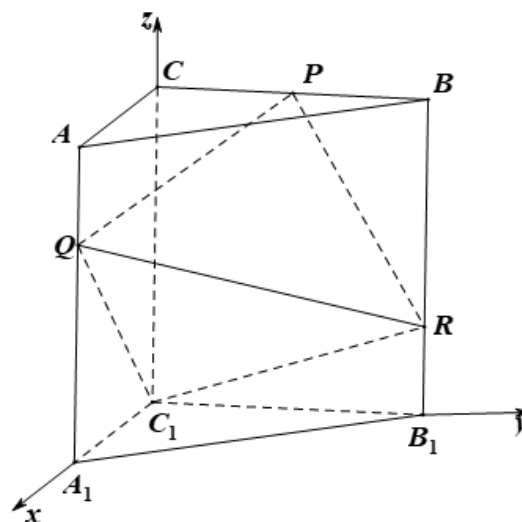


Figure 1. Schematic Diagram of the Modified Example 8

Request: (1) The cosine value of the angle formed by plane PQR and plane $A_1B_1C_1$;
 (2) Calculate the cosine value of the dihedral angle $P-QR-C_1$.

After the students finished their solutions, they raised a very natural but difficult new question that could not be answered directly and clearly: How can we use the normal vector of a plane to determine the cosine value of the dihedral angle? Unlike the angle between a line and a plane (where the cosine value is positive and does not exceed 90°), the size of the dihedral angle can be greater than 90° , and the cosine value can be negative. The key to solving this problem is to determine the direction of the normal vector of the plane we are seeking. This question triggered Teacher D's thinking.

2.3. Research Methods

Drawing on the methods of case study and action research [19,20], in order to address the real problems occurring in the classroom, a regional high school-university mathematics bridging network community was established, as shown in Figure 2 [21,22]. The members of this network community include high school mathematics teachers with the intention of improving their professional qualities, high school mathematics curriculum coordinators, and university mathematics teachers [23]. For the practical problems encountered by high school mathematics teachers, activities such as textbook study, curriculum standard interpretation, lesson case development, and lesson evaluation are carried out irregularly. During these activities, mutual benefit and growth are achieved. University mathematics teachers understand the current situation of high school mathematics teaching, high school mathematics curriculum coordinators enhance their ability to combine theory with practice, and high school mathematics teachers improve their ability to interpret textbooks from a "high perspective" and their practical skills [24].

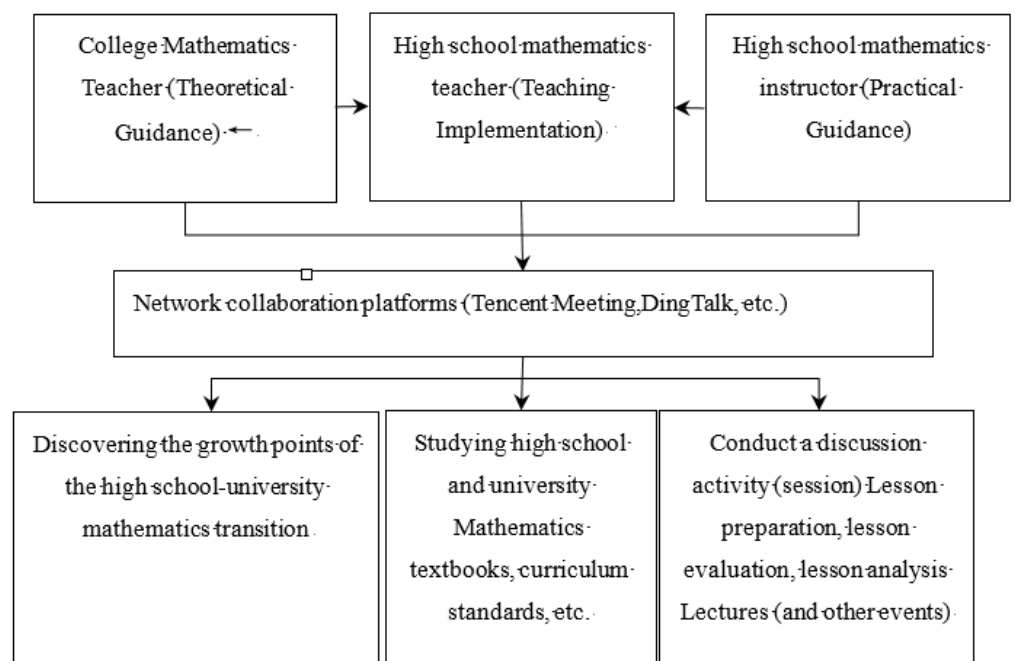


Figure 2. Network Teaching and Research Community Structure of High School Mathematics and University Mathematics Teachers.

Research on the online teaching and learning community for high school and university mathematics should focus on the following aspects: First, interactive interpretation of textbooks. For the teaching contents presented in both high school and university mathematics textbooks, with the guidance of subject experts and teaching researchers, teaching design and practice are carried out to explore the mathematical ideas

contained in the university mathematics textbooks, and then adapt them to the high school level and integrate them into high school mathematics classroom teaching. Second, mutual observation of teaching methods. Through the network platform, high school and university mathematics teachers observe each other's lessons on the same main content, understand each other's teaching methods, so as to better align their teaching methods. Third, mutual improvement in high school and university mathematics teaching. Through mutual communication, high school and university mathematics teachers leverage their respective strengths in theory and practice to solve common problems and overcome difficulties in the high school-university mathematics connection. Fourth, developing cases for the high school-university mathematics connection.

3. Research Results

3.1. Lesson Plan Development Pathway

In order to answer the questions raised by the students, Teacher D sought advice from the school's teaching research group and the teaching research community. Eventually, it was decided to integrate the content of the selective compulsory textbooks of the People's Education Edition (2004) and the People's Education Edition (2019), and propose solutions for elementary mathematics and advanced mathematics. Through communication and discussion, a consensus was reached: the curriculum standards, textbooks, and the college entrance examination have all weakened the requirements for in-depth understanding and calculation of the concept of dihedral angles. Highly abstract methods, such as constructing auxiliary vectors, are presented in the form of reading materials and placed before class for students to read independently. Compared to problem-solving itself, exploring the use of "high-level perspectives" to solve high school mathematics problems and introducing vector cross products and the connection between high school and university mathematics are more meaningful. The vector cross product not only simplifies the method of constructing auxiliary vectors but also greatly helps stimulate students' desire for autonomous inquiry learning. Thus, the lesson case development path as shown in Figure 3 was constructed.

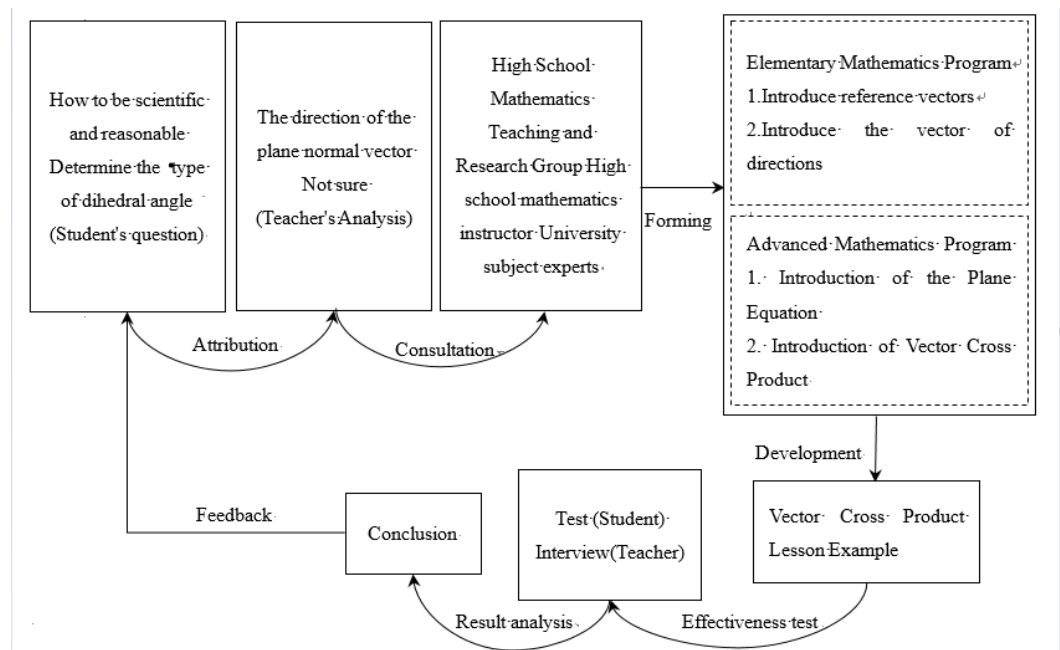


Figure 3. The Development Path for Judging the Direction of the Plane Normal Vector

3.2. Teaching Design for Vector Cross Product

Firstly, the study examines the 2017 version of the curriculum standards, the 2020 revision of the standards, and the corresponding textbooks to clarify the selection of

teaching content. The study selects the high school mathematics Elective A courses, where the mathematics and physics courses include the topic "Space Vectors and Algebra." The requirements for vector cross product in this topic are stated as: explore and explain the content of vector cross product and its geometric meaning. By referring to the elective course textbooks and university mathematics related textbooks, it is found that the course content mainly includes: the concept, properties, operational rules, coordinate representation formula, and applications of vector cross product. The coordinate representation of vector cross product can simplify the content of the compulsory courses and use a system of three linear equations to solve the normal vector of a plane, while providing the specific direction of the normal vector, making it easier for students to understand the situation where there is a minus sign when calculating the normal vector of a plane, laying the foundation for the determinant representation of the coordinates of the normal vector. The geometric meaning of vector cross product can be used to determine the sign of the cosine value of the dihedral angle, and its modulus can be linked with the knowledge of solving triangles. Using vector cross product instead of scalar product is more convenient for proving the sine theorem and deriving the Heron formula, which is a manifestation of the specific application of vector cross product. Therefore, the teaching research team decided to incorporate the application of vector cross product in plane geometry, and designed a total of 2 example problems.

Secondly, the corresponding courses of university mathematics were analyzed, and the teaching objectives and teaching design process were formulated. In the preparation stage of teaching design, four university mathematics textbooks were mainly referred to, namely, "Analytic Geometry" edited by Qiu Weisheng, "Analytic Geometry" edited by Lu Lingen and Xu Zidao, "Advanced Mathematics" edited by the Mathematics Department of Tongji University, and "Advanced Algebra and Analytic Geometry" edited by Chen Zhijie. After careful thinking and discussion among the teaching research community, the final teaching objectives were determined: to master the definition and properties of vector cross product; to understand the derivation process of the coordinate representation of vector cross product and the determinant representation; to be able to apply vector cross product to solve specific problems in solid geometry and plane geometry. To achieve the teaching objectives, the teaching adopts a problem-driven approach and emphasizes student thinking, emphasizing the design concept of revealing the essence of knowledge and the connection between previous and subsequent knowledge. Relying on the real problems encountered by students, it guides students to understand the necessity of learning vector cross product, then constructs the connection between vector cross product and previously learned knowledge, and reflects the instrumental nature of vector cross product. Finally, the teaching design process shown in Figure 4 was determined.

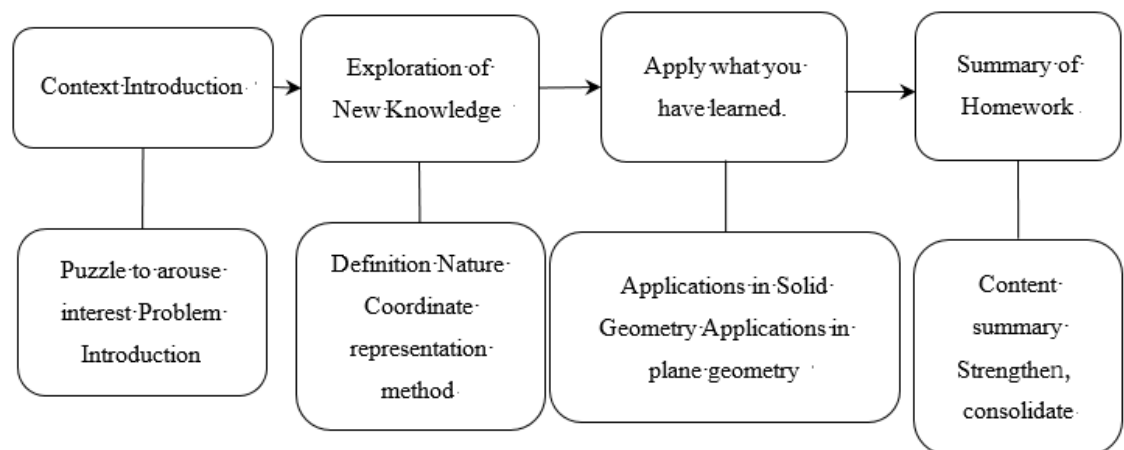


Figure 4. Flowchart of Teaching Design for Vector Cross Product

In the context-setting stage, through the real problems encountered in classroom teaching-the determination of the sign of the cosine value of the dihedral angle-questions are raised to arouse students' interest, the necessity of learning new knowledge is revealed, and students' interest in learning and the desire to explore new knowledge are stimulated. This lasts for about 5 minutes.

In the new knowledge exploration section, the concept, properties and coordinate representation of vector cross product are introduced. This includes the introduction of second-order determinants and third-order determinants, and this section takes approximately 35 minutes.

The application section involves introducing the application of vector outer product in solid geometry and plane geometry. It explains the relationship between the direction of vector outer product and the normal vector of a plane, completely resolving the determination of the sign of the cosine value of the dihedral angle; it also clarifies the intrinsic connection between the modulus of vector outer product and the knowledge of solving triangles, applying vector outer product to calculate the area of spatial triangles and deriving the sine theorem. This section takes approximately 30 minutes.

In the summary and homework session, students independently exchange their learning gains, while the teacher provides supplementary comments and summaries. Additionally, three after-class practice questions are assigned to enable students to consolidate their knowledge of vector cross product by solving problems.

Exercise 1 As shown in Figure 5, in the right triangular prism $ABC-A_1B_1C_1$, AB is perpendicular to AC , $AB=AC=2$, and $AA_1=4$. D and E are the midpoints of AB and BC respectively, satisfies $\overrightarrow{A_1F} = \frac{1}{3}\overrightarrow{A_1C}$. Calculate the cosine value of the dihedral angle $A-DF-E$.

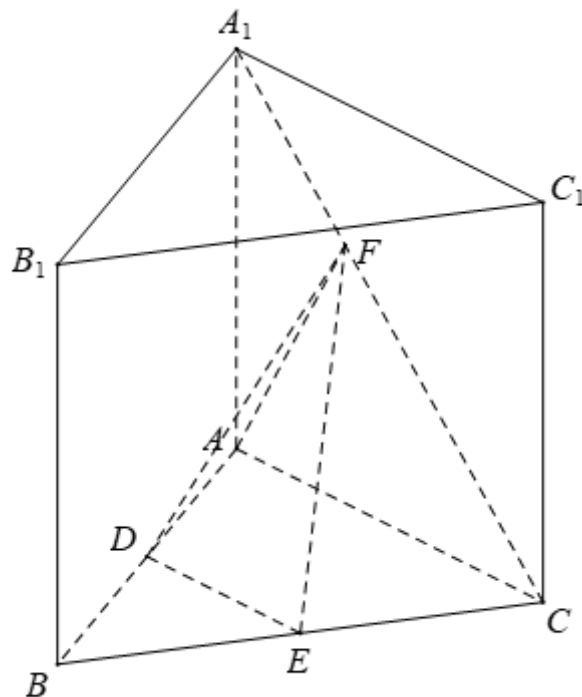


Figure 5. Right Triangular Prism Used in Exercise 1

Exercise 2. Using the operation rules of vector product, prove that: $(a - b) \times (a + b) = 2(a \times b)$.

Exercise 3. Prove the Heron formula for the area of a triangle using vector methods.

3.3. Key Segments of Teaching Implementation

The key teaching steps are as follows:

Teacher: During the previous teaching period, a student raised an excellent question: How can we determine the sign of the cosine of a dihedral angle? After studying the pre-class reading materials, everyone should have a solution. Can you come up with a simpler method to solve the above problem? We have learned that a point on a plane and a normal vector of the plane can uniquely determine a plane. Students who think diligently might ask: Is there a method that can directly determine the direction of the normal vector of a plane?

Teacher: The answer is yes. Today, we will learn a new operation on vectors and try to solve the problems we encountered earlier. We will also gain an understanding of the application of vector algebra in solid geometry.

Teacher: By analogy with the vector dot product (referred to as the inner product), define the vector cross product (referred to as the outer product) of vectors. The outer product of vectors a and b . It is denoted as $a \times b$, and the result is a vector. $|a \times b| = |a| |b| \sin \angle a, b$, the outer product is a vector. How to determine the direction?

Teacher: Extend your right hand, and let the four fingers point in the direction of the first vector. Then, rotate clockwise to the second vector. Big. The direction pointed by the thumb is the direction of the cross product of vectors a and b . The teacher and students jointly determine the direction of the vector cross product as shown in Figure 6.



Figure 6. Determination of the Direction of the Plane Normal Vector in the Teaching Site

Teacher: Is the direction of $a \times b$ perpendicular to the plane determined by vectors a and b ?

The direction of the product $c = a \times b$ is determined according to the right-hand rule, by turning from vector a to vector b at an angle no greater than π .

Student G1: Yes, $a \times b$ is a normal vector of the plane determined by a and b .

Property: $a \times a = 0$ The necessary and sufficient condition for $a \parallel b$ to be true is $a \times b = 0$.

The vector product follows the following operational rules

1. Inverse Commutativity: $a \times b = -(b \times a)$
2. Regarding the associative law of scalar multiplication: $\lambda(a \times b) = (\lambda a) \times b = a \times (\lambda b)$
3. Distributive Law: $(a \times b) \times c = a \times c + b \times c$

Teacher: When we solve practical problems, we need the coordinates of the normal vector of the plane. How can we represent the coordinates of the normal vector of the plane? ! topic Student G2: It is necessary to know the coordinate representation of the cross product of two vectors

Teacher: Introduce the concept of second-order determinants. $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$, $a_{ij} \in R$ ($i, j = 1, 2$), as shown in Figure 7, interested students can refer to and study the high school mathematics elective textbooks or university advanced mathematics textbooks for further understanding. (Provide the definition and representation of the

third-order determinant, clarify the doubts about positive and negative values, and incidentally hint at the properties of the determinant.)

Question 10: The result of the vector cross product is a vector. How can the vector cross product be represented in coordinates?

二阶行列式的概念, $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$, 其中 $a_{ij} \in \mathbb{R}$ ($i, j=1, 2$)

平面 α 上不共线的向量 $\vec{a} = (x_1, y_1, z_1)$, $\vec{b} = (x_2, y_2, z_2)$, 则

$$\vec{a} \times \vec{b} = \begin{pmatrix} y_1 z_2 - z_1 y_2 \\ z_1 x_2 - x_1 z_2 \\ x_1 y_2 - y_1 x_2 \end{pmatrix}$$

Figure 7. Teaching Scene of Second-Degree Determinant

Teacher: Try to solve the example using vector cross product, as shown in Figure 8.

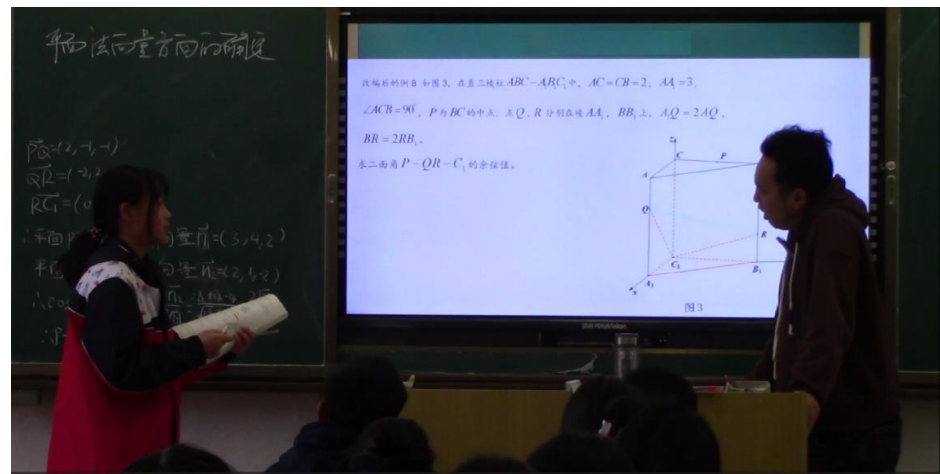


Figure 8. Illustrative Examples and Their Solutions

The solution process is written as follows: (1) Use the determinant form of the vector cross product to find the normal vectors of the two planes; (2) use the direction of the cross product to orient the planes and determine whether the two normal vectors are in opposite directions or in the same direction; (3) calculate the cosine value of the dihedral angle.

Teacher: Now, I will introduce the application of vector cross product in plane geometry. By connecting to the knowledge we have learned before, let's appreciate the role of vector cross product as a tool.

The geometric meaning of the magnitude of the vector cross product: If two vectors are not collinear, their magnitude represents the area of the parallelogram formed by the two vectors as adjacent sides, and this area is twice the area of the triangle formed by the two vectors. What other theorems or conclusions related to triangle area are there? Or what do you think of when you see $ab \sin \theta$?

Example 1 Given three points $A(1,2,3)$, $B(3,5,4)$, and $C(6,8,7)$ in space, find the area of triangle ABC.

Solution: According to the definition of the vector product, it can be known that the area of triangle ABC, $S_{\triangle ABC} = \frac{1}{2} |\vec{AB}| |\vec{AC}| \sin \angle A = \frac{1}{2} |\vec{AB} \times \vec{AC}|$. Since $\vec{AB} = (2,2,2)$ and $\vec{AC} = (5,6,4)$, therefore $\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 2 & 2 & 2 \\ 5 & 6 & 4 \end{vmatrix} = 4i - 6j + k$, thus $S_{\triangle ABC} = \frac{1}{2} |4i - 6j + k| = \frac{1}{2} \sqrt{4^2 + (-6)^2 + 1^2} = \frac{\sqrt{53}}{2}$.

Question the students: Given the coordinates of three points in the Cartesian coordinate system, how to calculate the area?

Example 2. Proving the sine theorem of a triangle $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Proof: As shown in Figure 9, based on the geometric properties defined by the vector product, the area of triangle ABC , $S_{\triangle ABC} = \frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2} |\vec{BC} \times \vec{BA}| = \frac{1}{2} |\vec{CA} \times \vec{CB}|$. Therefore, $|\vec{AB}| \cdot |\vec{AC}| \sin \angle A = |\vec{BC}| \cdot |\vec{BA}| \sin \angle B = |\vec{CA}| \cdot |\vec{CB}| \sin \angle C$. That is to say $cb \sin A = ac \sin B = ab \sin C$, Splendid abc , $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

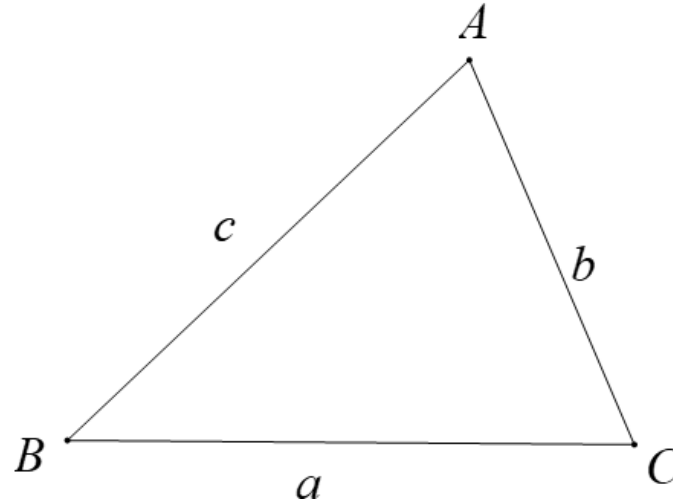


Figure 9. Schematic Diagram of the Triangle in Example 2

3.4. Evaluation and Reflection of Teaching Effectiveness

After the above classroom teaching was completed, the teaching research team further examined the students' feedback and the effectiveness of teaching and learning through questionnaires, tests and interviews. The questionnaire consisted of six questions (see Appendix A). It was designed around four dimensions: determination of the direction of the normal vector of a plane, the mathematical ideas contained therein, the distinction between the dihedral angle and the angle between two planes, and the application of the direction of the normal vector of a plane. The survey results of the questionnaire are shown in Figure 10.

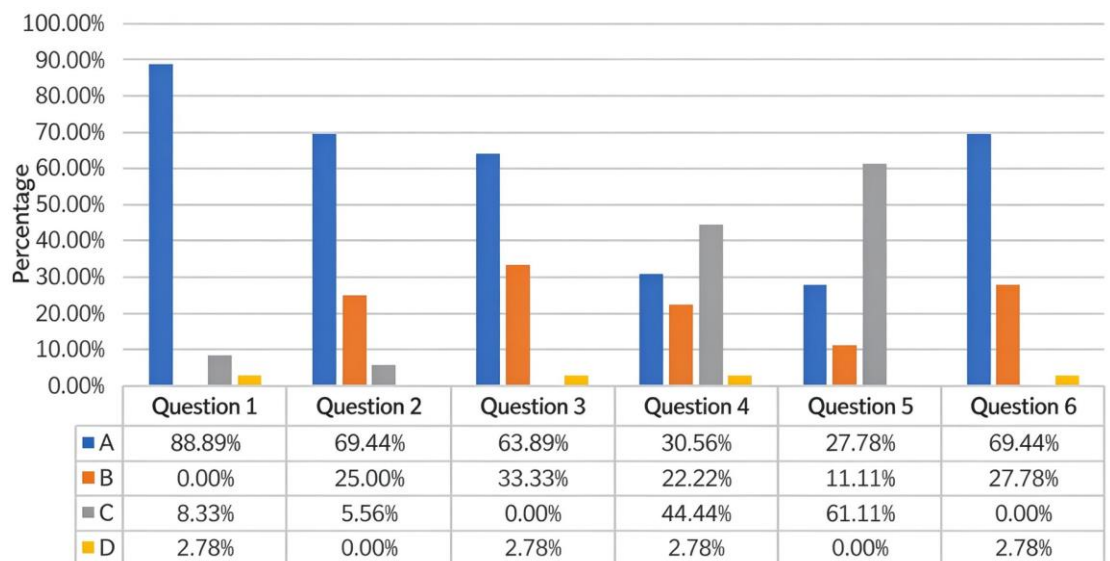


Figure 10. Effect of Questionnaire Survey

Analyzing the student questionnaire survey, the teaching effect was quite remarkable. The results of Question 1 and Question 2 showed that 13.67% of the students were unable to find the normal vector of a plane or could not correctly determine the direction of the normal vector of a plane; the result of Question 3 indicated that 97.22% of the students had mastered or basically mastered the mathematical idea of determining the direction of the normal vector of a plane using the vector cross product; the results of Question 4 and Question 5 indicated that nearly 40% of the students could distinguish between the concepts of dihedral angles and the angles between planes; the result of Question 6 showed that 69.44% of the students could correctly apply the plane normal vector to accurately calculate the cosine value of the dihedral angle.

In order to further test students' understanding and application of knowledge, two test questions were adapted from the textbook exercises and the college entrance examination questions, and a scoring standard was formulated (see the Appendix B). After class, the students took the test for 20–30 minutes. This was to further understand students' mastery and application of the direction of plane normal vectors. Question 1 mainly tested students' understanding of the direction of plane normal vectors, while Question 2 aimed to test students' simple application of the direction of plane normal vectors. The test results are shown in Figure 11.

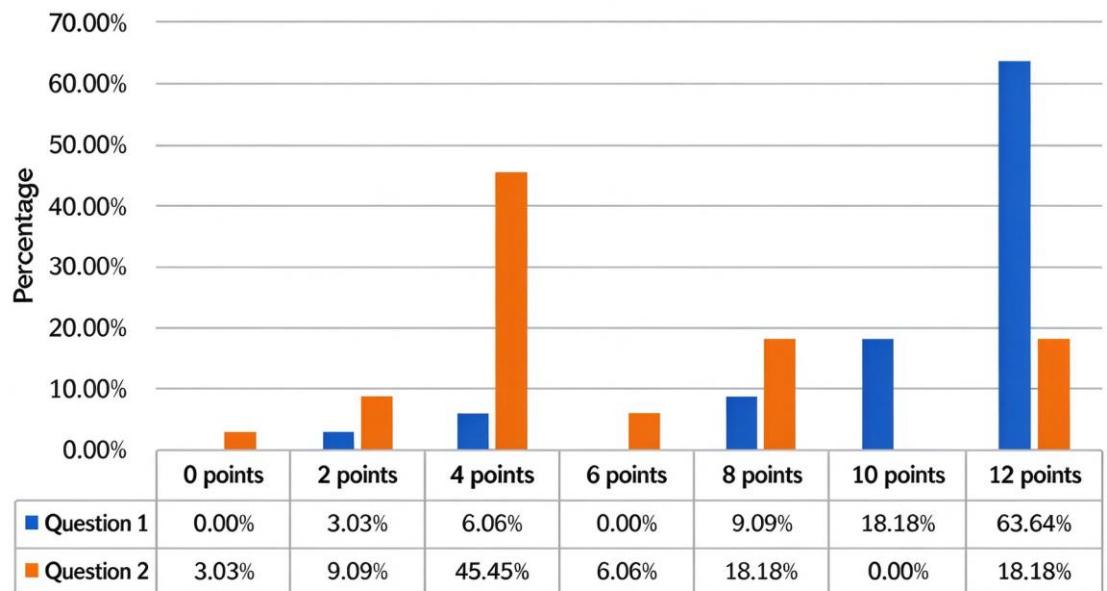


Figure 11. Statistical Chart of Students' Learning Effectiveness Test Data

From the overall analysis in Figure 11, both Question 1 and Question 2 are designed to test students' ability to solve dihedral angles using the normal vector of a plane. By comparing the student response data of Question 1 with that of Question 2, it can be seen that the overall correct response rate of Question 1 is generally higher than that of Question 2. From the test results of Question 1, it can be concluded that 63.64% of the students were able to correctly calculate and determine the type of dihedral angle based on the direction of the plane normal vector. 18.18% of the students could perform the calculation correctly but were unable to accurately determine the direction of the plane normal vector. Overall, the students' understanding of the direction of the plane normal vector was significantly improved. Question 2 showed that students could apply the direction of the plane normal vector to solve simple dihedral angles with parameters, but the results were not satisfactory. Only 18.18% of the students were able to complete the solution successfully. Further teaching should strengthen the specific application of vector cross product in the solution of dihedral angles.

In addition to questionnaires and tests, individual interviews with students and classroom observations revealed that although the teaching content covered knowledge of second-order and third-order determinants in linear algebra, it was novel and easy for students to understand, which in turn stimulated their curiosity and achieved teaching effects beyond the expected ones. Given the initial attempt at the practice of connecting high school and university mathematics, the teaching content needs to be further developed and improved. For example, after teaching the concept of vector cross product, the difficulty of coordinate representation should be reduced, guiding students to explore and familiarize themselves with $i \times j = k$, $j \times k = i$, $k \times i = j$, as well as the results of operations after changing the order, to adapt to the cognition of more students and facilitate better understanding by students.

4. Conclusion

4.1. Cooperation between High Schools and Universities: Creating a Learning Community

As mentioned earlier, the cooperation model between high schools and universities that focuses on the cultivation of a few outstanding students is more dependent on university mathematics teachers. Usually, university mathematics teachers go to high schools to give individual lessons to selected students, and higher requirements are imposed on the teachers. However, it cannot be denied that such bridging measures have significant value. In the context of the popularization of higher education, high school-university mathematics bridging efforts for a wider range of high school students should appropriately weaken the systematicity and depth of university mathematics learning. Such forms of high school-university bridging should be led by high school mathematics teachers, because high school mathematics teachers are more familiar with regular teaching and students' learning situations. However, existing studies have shown that high school mathematics teachers have various difficulties and limitations when integrating university mathematics knowledge into regular teaching. In this case, creating a high school-university cooperative learning community can effectively solve this problem, and such cooperation is also easy for university mathematics teachers to carry out. University mathematics teachers do not need to spend an entire period of time teaching in middle schools; instead, they offer suggestions for the problems encountered by high school mathematics teachers and provide materials and suggestions from a university perspective. With the support of the high school-university cooperative learning community, high school mathematics teachers can also improve their professional literacy and enhance their willingness and ability to conduct high school-university mathematics bridging teaching. Thus, it is possible to carry out inclusive embedded bridging teaching under the limited conditions of high school mathematics teaching, develop students' core competencies and professional interests, and prepare them for university mathematics learning.

4.2. Jointly Building and Sharing: Developing the Lesson Case Resource Library

University mathematics teachers, whether they are instructors for general mathematics courses or those specializing in mathematics, have accumulated a lot of personal experience during their long-term teaching careers regarding the difficulties encountered by university freshmen in their mathematics learning. They have also published relevant materials. This part of the content serves as reference material for the development of high school-to-university mathematics bridging lessons. Of course, most books and articles on bridging issues are in the form of knowledge points and exercises, lacking in-depth discussion, and have limited guiding and reference value for teachers' classroom teaching. However, the study, discussion, and sharing of these materials can serve as a valuable resource library for the development of high school-to-university mathematics bridging lesson cases. The high school-to-university bridging in mathematics education has a specific background and special significance. Klein proposed to view elementary mathematics from a higher perspective very early on, and wrote a series of books during the training process for high school mathematics teachers.

Because these books were written by renowned mathematicians, high school mathematics teachers may have some difficulty in reading them. In such cases, high school and university teachers can read and share their reading experiences together, which is beneficial for high school mathematics teachers' understanding and for training future university teachers. Some journals focusing on higher mathematics teaching, such as *Research on Advanced Mathematics*, can also be included as auxiliary learning resources for lesson case development.

4.3. Evidence-Based Iteration: Facilitating the Transition from High School to University

To overcome common problems such as subjective arbitrariness and weak operability in lesson case development, the development of inclusive embedded high school-university mathematics bridging lesson cases should be more evidence-based. It can draw on the concepts of evidence-based clinical practice to improve the detailed operational procedures for developing evidence-based high school-university bridging lesson cases. Based on design research, the lesson cases can be continuously iterated and refined to make them more convenient for high school mathematics teachers to reference and use. The four basic stages mentioned in the text—"preparation-development and design-teaching implementation-evaluation and feedback"—are consistent with the operational steps of unit teaching design. In fact, the inclusive embedded high school-university mathematics bridging lesson example focuses on a university knowledge topic closely related to high school mathematics. It usually requires 2–3 class hours. This micro-topic course can also be regarded as a special type of unit teaching. From the perspective of unit teaching design, optimizing the development of such lesson examples is worthy of further research. In addition, more attention should be paid to investigating students and collecting their opinions. Under certain conditions, strengthening follow-up research is necessary to track whether high school students can activate relevant cognition when they learn the content covered by the bridging lesson example after entering university. Strengthening the development of high school-university bridging lesson examples can promote empirical research on the professional development of high school mathematics teachers. It can also provide references and suggestions for effective teacher training and for implementing the policy of integrating excellence and fairness in high school-university bridging education.

Appendix A. Determination of the Direction of Plane Normal Vectors (Student Questionnaire)

Dear students, hello! For the purpose of mathematics teaching research, we are conducting a survey. We sincerely ask you to fill out the questionnaire truthfully based on your learning experience. Your support is the continuous driving force for our research. This survey is anonymous and will not involve personal privacy. The results will only be used for research and will not be released publicly. Thank you very much for your cooperation!

- 1) Can you determine the direction of a plane normal vector? ()
A. Yes B. No C. Uncertain D. Not mastered
- 2) Do you understand how to determine the direction of a plane normal vector? ()
A. Fully understand and apply correctly B. Fully understand but make mistakes
C. Poorly understand D. Do not understand at all
- 3) Have you grasped the mathematical thinking for determining the direction of plane normal vectors? ()
A. Mastered B. Basically mastered C. Uncertain D. Not mastered
- 4) Is there a difference between the concepts of "dihedral angle" and "angle between two planes"? ()
A. No B. There is a difference but cannot state it clearly C. There is a difference and know the specific distinction D. Unaware of these two concepts
- 5) Is the angle between the normal vectors of two planes related to the plane angle of the dihedral angle formed by the two planes? ()

- A. No B. Related and equal in magnitude C. Related and either equal or supplementary in magnitude D. Uncertain
- 6) Can you accurately calculate the plane angle of the dihedral angle of two planes using the angle between their normal vectors? ()
- A. Yes and can get correct calculation results B. Yes but make mistakes in the sign of calculation results C. Uncertain D. No

Appendix B. Test Questions and Scoring Criteria for Determining the Direction of Planar Normal Vectors

1. As shown in the Figure 12, in the right triangular prism $ABC-A_1B_1C_1$, AB is perpendicular to AC , and $AB=AC=2$, $AA_1=4$, D, E , Let F be the midpoints of AB, BC , and A_1C . Find the cosine value of the dihedral angle $A-DF-E$.

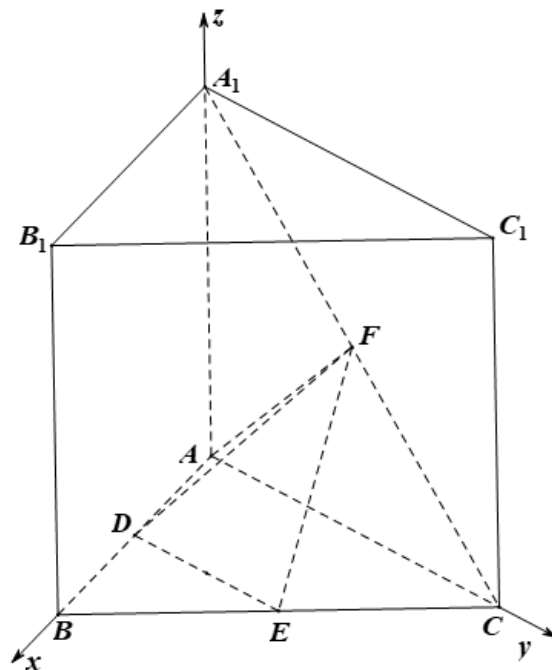


Figure 12. Right Triangular Prism for Exercise 1

Solution: In a right triangular prism $ABC-A_1B_1C_1$, $AB \perp AC$.

Take point A as the origin of the space rectangular coordinate system, and establish the space rectangular coordinate system as shown in the figure with lines AB, AC, AA_1 as the x, y, z -axes respectively. 2 points

$$\because AB=AC=2, AA_1=4$$

$$\therefore A(0,0,0), D(1,0,0), E(1,1,0), F(0,1,2)$$

$$\therefore \overrightarrow{AD}=(1,0,0), \overrightarrow{AF}=(0,1,2), \overrightarrow{DE}=(0,1,0), \overrightarrow{DF}=(-1,1,2) \dots \dots \dots 4 \text{ points}$$

$$\therefore \overrightarrow{AD} \times \overrightarrow{AF}=(0,-2,1), \overrightarrow{DE} \times \overrightarrow{DF}=(2,0,1)$$

$\therefore$ A normal vector of the plane ADF is $\vec{n}_1=(0,-2,1)$, a normal vector of the plane DEF is $\vec{n}_2=(2,0,1) \dots \dots \dots 6 \text{ points}$

$$\therefore \cos \langle \vec{n}_1, \vec{n}_2 \rangle = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{1}{5} \dots \dots 8 \text{ points}$$

Let the plane angle of the dihedral angle $A-DF-E$ be θ , then $\theta = 180^\circ - \langle \vec{n}_1, \vec{n}_2 \rangle \dots \dots \dots 10 \text{ points}$

The cosine value of the dihedral angle $A-DF-E$ is $-\frac{1}{5} \dots \dots \dots 12 \text{ points}$

2. As shown in the Figure 13, in the regular square prism $ABCD-A_1B_1C_1D_1$, where $AB=2$ and $AA_1=4$. Points A_2, P, C_2, D_2 respectively On the edges AA_1, BB_1, CC_1, DD_1 , we have $AA_2=1, CC_2=3, DD_2=2$. If the dihedral angle $P-A_2C_2-D_2$ is 150° . Find B_1P .

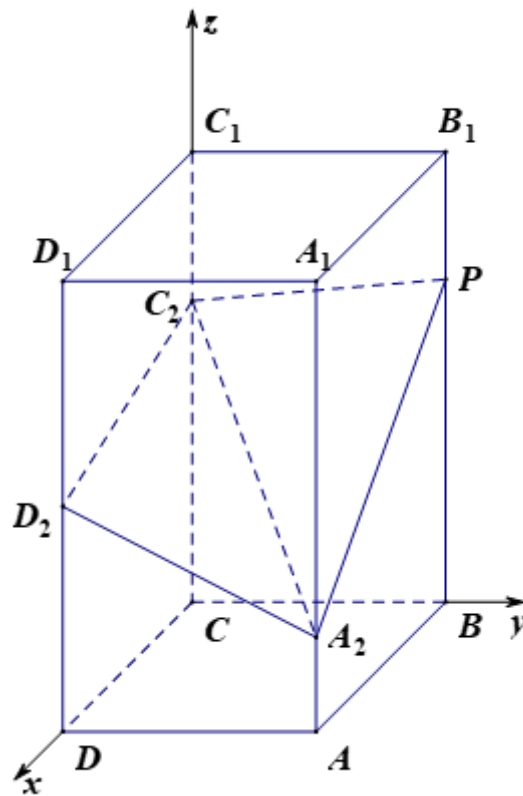


Figure 13. Regular Square Prism for Exercise 2

Solution: $\because ABCD-A_1B_1C_1D_1$ is a regular quadrilateral.

$\therefore$ Taking point C as the origin of the three-dimensional Cartesian coordinate system, the lines CD, CB and CC_1 respectively Establish a three-dimensional Cartesian coordinate system as shown in the figure for the x, y, and z axes. C -xyz..... 2 points

From the problem statement, we know: $A_2(2,2,1)$, $D_2(2,0,2)$, $C_2(0,0,3)$, $P(0,2,a)$ 4 points.

$\therefore \overrightarrow{A_2D_2} = (0, -2, 1)$, $\overrightarrow{A_2C_2} = (-2, -2, 2)$, $\overrightarrow{A_2P} = (-2, 0, a-1)$ 6 points

$\therefore \overrightarrow{A_2C_2} \times \overrightarrow{A_2D_2} = (2, 2, 4)$, $\overrightarrow{A_2P} \times \overrightarrow{A_2C_2} = (2a-2, 6-2a, 4)$

$\therefore$ A normal vector for plane $A_2C_2D_2$ is $\vec{n}_1 = (1, 1, 2)$, and a normal vector for plane A_2C_2P is $\vec{n}_2 = (a-1, 3-a, 2)$ 8 points

$\therefore$ The dihedral angle $P-A_2C_2-D_2$ is 150°

$\therefore \langle \vec{n}_1, \vec{n}_2 \rangle = 30^\circ$

$\therefore \cos \langle \vec{n}_1, \vec{n}_2 \rangle = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{\sqrt{3}}{2}$ 10 points

$\therefore a=1$ or $a=3$

$\therefore B_1P=1$ or $B_1P=3$...12 points

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