

Article

Multi-Aircraft Prescribed Time Formation Control

Fengning Zhang^{1,*}

¹ College of Physics and Electronic Information Engineering, Jining Normal University, Wulanchabu, China

* Correspondence: Fengning Zhang, College of Physics and Electronic Information Engineering, Jining Normal University, Wulanchabu, China

Abstract: A prescribed time control strategy is proposed to meet the prescribed time convergence performance requirement of cooperative errors in multi-aircraft formation. The mathematical models of the dynamics and kinematics of multi-aircraft, and the formation error control model are developed. A sliding mode control approach with the prescribed time convergence is designed to enhance the convergence of cooperative formation errors of multiple aircraft under uncertainties, in order to satisfy high-performance demands of cooperative control of multiple aircraft. Moreover, strict evidence of multi-aircraft prescribed time formation is given on the basis of Lyapunov stability theory.

Keywords: Prescribed-Time Stability; Nonlinear Systems; Robot Control; Sliding Mode Control; Switched Control

1. Introduction

Formation control of flight vehicles can fully exploit the benefits of multiple flight vehicles and achieve tasks that cannot be achieved by a single flight vehicle. In multi-flight-vehicle formation, controllers may be designed based on asymptotic convergence control methods or finite-time convergence control methods. The literature proposes machine learning-assisted adaptive control of spacecraft formation path planning and collision avoidance [1]. This paper is an introduction to a machine learning-based adaptive control method of trajectory planning and collision avoidance in multi-spacecraft formation flying (multi-SFF). The proposed algorithm is a combination of a radial basis function neural network (RBFNN)-aided adaptive nonlinear feedback controller used in path planning with a fuzzy-learning-aided adaptive collision avoidance (FACA) scheme, and is known as the RBFNN&FACA algorithm. Lyapunov stability analysis and elaborate fuzzy reasoning are used to confirm the reliability and effectiveness of the method. It is demonstrated that the RBFNN-assisted control drives the system to the desired state at an exponentially faster rate while rapidly suppressing steady-state oscillations. FACA component presents a new collision probability distribution function which uses relative velocity information in making avoidance decisions thus addressing the shortcomings of traditional artificial potential field (APF) approaches. Further study shows that FACA can be reversed and does not rely on the path planning controller and this simplifies the overall control architecture and indicates possible use in noncooperative orbital games between spacecraft. Moreover, a novel time-dependent operator is presented to convert the nonlinear multi-SFF dynamics into a simplified equivalent form without loss of accuracy. Numerical simulations of a circling configuration of five spacecraft under J2 perturbation and virtual disturbances of Earth are performed. The proposed method demonstrates a considerable improvement in time response, nonlinearities and disturbance robustness, and autonomous performance in complex situations compared to APF-based ones. Overall, this work offers a practical source of information about the use of machine learning in controlling spacecraft. The paper [2] discusses adaptive optimal formation tracking control of a group of spacecraft with control input constraints and entirely unknown dynamics. Optimal coordination

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with uncertain system models is very difficult in real space missions like debris removal and cooperative observation. To address this challenge, a distributed adaptive control structure is created through differential graphical games and a low-gain feedback method. A parameterized algebraic Riccati equation (ARE) with a tunable low-gain parameter results in a family of distributed optimal control policies that do not saturate the inputs. Furthermore, a model-free adaptive dynamic programming algorithm founded on value iteration is implemented to compute the parameterized ARE without prior information of the spacecraft dynamics. Adaptive coupling gains are also provided to ensure closed-loop stability. Strict asymptotic stability analysis is given, and simulation outcomes prove the usefulness of the suggested approach in the cases of spacecraft formation tracking. The current paper [3] examines the possibility of using Lorentz force and aerodynamic force together as a propellantless control mechanism of spacecraft formation flying. It is assumed that each spacecraft has several large flat plates that can rotate to create aerodynamic force, and Lorentz force is generated by varying the electrostatic charge of the spacecraft. An adaptive output feedback controller is designed with a sliding mode observer to manage unknown uncertainties and absence of relative velocity measurements. An optimal control allocation scheme is proposed to calculate the necessary electrostatic charge on the Lorentz spacecraft and the commands to the atmospheric actuators to distribute the control input effectively. Numerical simulations are carried out to ensure the effectiveness of the proposed control strategy in the presence of J2 perturbations. The findings show that the propellantless control approach can precisely control relative motion in a formation despite the presence of uncertainties and the unavailability of velocity measurements. One of the major contributions of this work is the presentation of a propellantless formation control method which exploits both Lorentz and aerodynamic forces thus eliminating the necessity of a conventional propulsion system.

The paper [4] is considering the distributed formation control of leader-follower multi-agent systems, where relative position information and a prescribed performance control strategy are employed. Both first-order and second-order agent dynamics are taken into account. It takes a leader-follower structure whereby a group of agents that receive external inputs are identified as leaders to lead the rest of the agents such that the entire system attains the required formation and meets specified transient performance constraints. Assuming that the interaction graph is a tree, a distributed control law is constructed in the case of the first order, under the condition that the decay rate of the performance functions satisfies an appropriate bound. Two types of tree graphs which allow other follower nodes are also discussed. In the case of the second order, a backstepping-based distributed control scheme is developed to the leader group to drive the whole system to the target formation within the desired performance bounds. Lastly, some simulation examples are given to demonstrate the usefulness of the proposed method. The paper [5] studies a leader-follower formation control problem, where the leaders have a constant velocity, and the followers are aware of the desired bearing vectors in the target formation and can measure either displacement vectors or bearing vectors with respect to multiple neighboring agents. In both measurement cases, distributed control laws are formulated to attain the required formation despite the presence of unknown bounded uniformly continuous uncertainties in the dynamics of the followers. Besides the assumption that all agents have common local reference frames, the suggested control laws do not need any relative velocity information, bearing rate data and communication between agents. Barbalat lemma is used to determine the convergence of the target formation and simulation results are given to confirm the theoretical results. Sliding mode variable structure control theory is used to develop a new cooperative control framework of multi-missile formation flight, but with practical consideration that the missile speed cannot be actively controlled [6]. The problem of formation control is initially defined in an inertial coordinate system based on the relative position of the leader and the follower. Then an acceleration transformation is used to transform the model to the ballistic coordinate system resulting in a form that is

convenient to use in further controller design. On this model, sliding mode variable structure controllers are developed in the nominal case without disturbances and the perturbed case with disturbances. Lyapunov stability theory is used to analyze the stability of the formation control system. The suggested approach is contrasted with another method which needs controllable speed. Numerical simulations indicate that the proposed scheme has relative position errors comparable to those of the comparison method even at uncontrollable speeds and the outcomes also show the strength, flexibility, and adaptability of the proposed control strategy to formations.

In order to enhance the performance of formation control of multiple flight vehicles, multi-flight-vehicle formation controllers need to be designed using finite-time convergence control techniques. The article [7] discusses the practical challenges of controlling small fixed-wing unmanned aerial vehicles to perform finite-time formation tracking in the case of ambient wind disturbances. To deal with the natural aerodynamic response, feedback linearization is used to derive two linear models having similar disturbances which are the translational and rotational dynamics respectively. Then a multistage constructive design process is presented where external forces are considered as intermediate control inputs on which the desired thrust and attitude commands are based. Moreover, a novel composite control scheme is created that integrates a distributed observer and an adaptive finite-time formation tracking controller. It is important to note that the design and implementation of the controller do not presuppose global knowledge of the communication topology, bounds of uncertainties, or the control inputs of the leader. Closed-loop stability is determined by Lyapunov analysis. Simulation results are given to show the effectiveness of the proposed method. The literature [8] discusses the formation control issue of several unmanned aerial vehicles (UAVs) through the development of a finite-time control strategy using terminal sliding mode (TSM). The UAV in question is a quadcopter capable of vertical takeoff, whose dynamics can be modeled as cascaded loops of rotation and translation. In order to ensure that the approach is applicable to low-cost UAV platforms, the control torque is synthesized by an auxiliary rotational system, thus eliminating the necessity of direct measurements of attitude. A terminal sliding mode surface is developed and added to a finite-time formation control protocol (FTFCP), which produces the driving thrust of many UAVs linked via an undirected communication topology in the translational subsystem. As a solution to safe flight of the UAV group in cluttered environments, a pigeon-hierarchy-inspired obstacle avoidance protocol (PHOAP) is suggested. The avoidance scheme creates repulsive potential fields on the maneuvering target of the leader and follower UAV cluster independently by emulating the interactive hierarchy seen in homing pigeon flocks. The laws of collision avoidance based on pigeon homing behavior are then combined with the finite-time sliding mode formation protocol and the resulting control torque is cascaded into the attitude loop to create a single obstacle-avoidance cooperative control structure. The effectiveness of the proposed method is validated by simulation scenarios where several UAVs arrive at a desired formation and move around obstacles. Finite-time nonlinear control algorithm, using the state-dependent differential Riccati equation in a Lyapunov framework, is introduced to reconfigure satellites and maintain formation in low Earth orbit (LEO) with explicit consideration of actuator saturation constraints to maintain the system within safe operating boundaries [9]. The objective of the proposed method is to obtain the required relative position and velocity vectors in a specified time in spite of orbital perturbations and accurate formation control is guaranteed by employing a high-fidelity relative motion model, which includes all zonal harmonic terms and atmospheric drag, the largest environmental disturbance in LEO. An optimization algorithm is also formulated to determine the most fuel-efficient state-dependent coefficient (SDC) parameterization, using a hybrid genetic algorithm with simulated annealing to reduce energy use and enhance overall performance. Also, a sensitivity analysis is conducted to find the best SDC parameterization of different satellite reconfiguration maneuvers, such as radial, along-track, and cross-track adjustments with various baseline distances, providing insights into the impact of

different parameterizations on the accuracy and efficiency of maneuvers. The proposed finite-time controller is compared to other SDRE-based controllers and demonstrates lower error margins, and the input saturation constraint incorporated in the control system guarantees that the control system stays within safe operating limits, ultimately contributing to successful mission execution. The article [10] examines the distributed adaptive cooperative time-varying formation tracking guidance issue of a team of aerial vehicles intercepting a maneuvering target. In contrast to previous works, the cooperative guidance strategies are developed based on distributed time-varying formation tracking theory. First, the leader vehicle is constructed to achieve interception with a desired terminal attack angle, hence providing a reference trajectory, and uncertain dynamic terms and unknown maneuvers of the target are compensated. In the case of the follower vehicles, distributed cooperative time-varying formation tracking guidance laws are suggested, and adaptive distributed extended state observers are proposed to estimate the states and input information of the leader. Then a process is given of finding the guidance parameters of the cooperative guidance laws. The characteristics and stability of the suggested algorithm are then examined thoroughly. Lastly, simulation outcomes are given to prove the viability and usefulness of the achieved outcomes.

This paper suggests a way of attaining formation control of multiple flight vehicles at the prescribed time that can enhance the formation control performance of multi-flight-vehicle systems. The findings can be extensively used in practical applications like UAV formation flying and cooperative control of multiple flight vehicles to improve mission execution efficiency and cooperative control performance of multi-flight-vehicle systems, as well as technical support of related engineering applications.

2. System Model

The motion of the i -th aircraft in a multi-vehicle formation can be described by the following equation:

$$\begin{cases} \dot{V}_{mi} = -g\sin\theta_{mi} + a_{mx2i} + d_{mx2i} \\ \dot{\theta}_{mi} = \frac{1}{V_{mi}}(-g\cos\theta_{mi} + a_{my2i} + d_{my2i}) \\ \dot{\psi}_{Vmi} = \frac{-1}{V_{mi}\cos\theta_{mi}}(a_{mz2i} + d_{mz2i}) \\ \dot{x}_{mi} = V_{mi}\cos\theta_{mi}\cos\psi_{Vmi} \\ \dot{y}_{mi} = V_{mi}\sin\theta_{mi} \\ \dot{z}_{mi} = -V_{mi}\cos\theta_{mi}\sin\psi_{Vmi} \end{cases} \quad (1)$$

Where $[x_{mi}y_{mi}z_{mi}]$ denotes the position state of the i -th aircraft, V_{mi} , θ_{mi} , ψ_{Vmi} are the aircraft's velocity, track inclination angle, and azimuth angle, g is the gravitational acceleration, $[a_{mx2i}a_{my2i}a_{mz2i}]$ denotes the control input for the i -th aircraft, $[d_{mx2i}d_{my2i}d_{mz2i}]$ represents the disturbance acting on the i -th aircraft. Define:

$$\begin{cases} e_{x1i} = x_{mri} - x_{mi} \\ e_{x2i} = \dot{e}_{x1i} = \dot{x}_{mri} - \dot{x}_{mi} \\ \dot{e}_{x2i} = \ddot{e}_{x1i} = \ddot{x}_{mri} - \ddot{x}_{mi} \end{cases} \quad (2)$$

$$\begin{cases} e_{y1i} = y_{mri} - y_{mi} \\ e_{y2i} = \dot{e}_{y1i} = \dot{y}_{mri} - \dot{y}_{mi} \\ \dot{e}_{y2i} = \ddot{e}_{y1i} = \ddot{y}_{mri} - \ddot{y}_{mi} \end{cases} \quad (3)$$

$$\begin{cases} e_{z1i} = z_{mri} - z_{mi} \\ e_{z2i} = \dot{e}_{z1i} = \dot{z}_{mri} - \dot{z}_{mi} \\ \dot{e}_{z2i} = \ddot{e}_{z1i} = \ddot{z}_{mri} - \ddot{z}_{mi} \end{cases} \quad (4)$$

Then the formation error control model of the i -th aircraft is

$$\left\{ \begin{array}{l} \dot{e}_{x1i} = e_{x2i} \\ \dot{e}_{y1i} = e_{y2i} \\ \dot{e}_{z1i} = e_{z2i} \\ \dot{e}_{x2i} = \ddot{x}_{mri} - \cos\theta_{mi}\cos\psi_{vmi}a_{mx2i} + \sin\theta_{mi}\cos\psi_{vmi}a_{my2i} \\ \quad - \sin\psi_{vmi}a_{mz2i} - \cos\theta_{mi}\cos\psi_{vmi}d_{mx2i} \\ \quad + \sin\theta_{mi}\cos\psi_{vmi}d_{my2i} - \sin\psi_{vmi}d_{mz2i} \\ \dot{e}_{y2i} = \ddot{y}_{mri} + g - \sin\theta_{mi}a_{mx2i} - \cos\theta_{mi}a_{my2i} - \sin\theta_{mi}d_{mx2i} \\ \quad - \cos\theta_{mi}d_{my2i} \\ \dot{e}_{z2i} = \ddot{z}_{mri} + \cos\theta_{mi}\sin\psi_{vmi}a_{mx2i} - \sin\theta_{mi}\sin\psi_{vmi}a_{my2i} \\ \quad - \cos\psi_{vmi}a_{mz2i} + \cos\theta_{mi}\sin\psi_{vmi}d_{mx2i} \\ \quad - \sin\theta_{mi}\sin\psi_{vmi}d_{my2i} - \cos\psi_{vmi}d_{mz2i} \end{array} \right. \quad (5)$$

We also can obtain the control model for the i -th aircraft

$$\left\{ \begin{array}{l} \dot{x}_{1i} = f_{1i}(x_{1i}, x_{2i}) \\ \dot{x}_{2i} = f_{2i}(x_{1i}, x_{2i}) + b_i(x_{1i}, x_{2i})u_i + d_i(x_{1i}, x_{2i}, t) \end{array} \right. \quad (6)$$

$$\text{where } x_{1i} = (e_{x1i}, e_{y1i}, e_{z1i})^T = (x_{1i}^{(1)}, x_{2i}^{(1)}, x_{3i}^{(1)})^T, \quad x_{2i} = (e_{x2i}, e_{y2i}, e_{z2i})^T = (x_{1i}^{(2)}, x_{2i}^{(2)}, x_{3i}^{(2)})^T, \quad f_{1i} = \begin{pmatrix} e_{x2i} \\ e_{y2i} \\ e_{z2i} \end{pmatrix}, \quad f_{2i} = \begin{pmatrix} \ddot{x}_{mri} \\ \ddot{y}_{mri} + g \\ \ddot{z}_{mri} \end{pmatrix}, \quad b_i = \begin{pmatrix} -\cos\theta_{mi}\cos\psi_{vmi} & \sin\theta_{mi}\cos\psi_{vmi} & -\sin\psi_{vmi} \\ -\sin\theta_{mi} & -\cos\theta_{mi} & 0 \\ \cos\theta_{mi}\sin\psi_{vmi} & -\sin\theta_{mi}\sin\psi_{vmi} & -\cos\psi_{vmi} \end{pmatrix}, \quad u_i = \begin{pmatrix} a_{xm2i} \\ a_{ym2i} \\ a_{zm2i} \end{pmatrix}, \quad d_i = \begin{pmatrix} -\cos\theta_{mi}\cos\psi_{vmi} & \sin\theta_{mi}\cos\psi_{vmi} & -\sin\psi_{vmi} \\ -\sin\theta_{mi} & -\cos\theta_{mi} & 0 \\ \cos\theta_{mi}\sin\psi_{vmi} & -\sin\theta_{mi}\sin\psi_{vmi} & -\cos\psi_{vmi} \end{pmatrix} \begin{pmatrix} d_{mx2i} \\ d_{my2i} \\ d_{mz2i} \end{pmatrix}, \text{ and assume } \|d_i\| \leq \bar{d}_i.$$

3. Prescribed Time Formation Control for Multi-Aircraft

The multi-aircraft prescribed time formation control (MAPTFC) is designed as

$$u_{Gi} = u_{\sigma(t)i} \quad (6)$$

where

$$u_{\sigma(t)i} := - \left(\frac{\partial f_{1i}}{\partial x_{2i}} b_i \right)^{-1} \left(\frac{\partial f_{1i}}{\partial x_{1i}} f_{1i} + \frac{\partial f_{1i}}{\partial x_{2i}} f_{2i} + w_{\sigma(t)i} \right)$$

$$w_{\sigma(t)i} := (w_{\sigma_1(t)i}^{(1)}, w_{\sigma_2(t)i}^{(2)}, \dots, w_{\sigma_m(t)i}^{(m)})^T$$

$$\sigma_{ki}(t) : [0, +\infty) \rightarrow W_i := \{1, 2\}$$

$$w_{i^*i}^{(k)} := \frac{o_i^{(k)} k_{ki}}{j_{ki}} \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)^{2 - \frac{j_{ki}}{k_{ki}}} - \frac{2O_i^{(k)}}{R_{ki}} + \left(\left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i + \varepsilon_{ki} \right) \text{sgn}(s_{i^*i}^{(k)})$$

$$w_{i_*i}^{(k)} := \frac{o_i^{(k)} k_{ki}}{j_{ki}} \dot{x}_{ki}^{(1)2 - \frac{j_{ki}}{k_{ki}}} + \left(\left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i + \varepsilon_{ki} \right) \text{sgn}(s_{i_*i}^{(k)})$$

$$s_{i^*i}^{(k)} := o_i^{(k)} \left(x_{ki}^{(1)} - \frac{O_i^{(k)}}{R_{ki}} t^2 + 2O_i^{(k)} t - O_i^{(k)} R_{ki} \right) + \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)^{\frac{j_{ki}}{k_{ki}}}$$

$$s_{i_*i}^{(k)} := o_i^{(k)} x_{ki}^{(1)} + \dot{x}_{ki}^{(1) \frac{j_{ki}}{k_{ki}}}$$

$$o_i^{(k)} := r_{ki}^{\frac{j_{ki}}{k_{ki}}} (x_{ki}^{(1)}(0) - R_{ki} O_i^{(k)})^{\frac{j_{ki}-k_{ki}}{k_{ki}}}, \quad r_{ki} := \frac{j_{ki}}{t_{pi}^{(k)}(j_{ki}-k_{ki})}, \quad O_i^{(k)} := \frac{-r_{ki} x_{ki}^{(1)}(0) - \dot{x}_{ki}^{(1)}(0)}{2 - R_{ki} r_{ki}}$$

with $2 - R_{ki} r_{ki} \neq 0$, $0 < R_{ki} < t_{pi}^{(k)}$, where $t_{pi}^{(k)}$ is the prescribed system convergence time and R_{ki} is the switching time, j_{ki} and k_{ki} are positive odd integers satisfying $1 < j_{ki}/k_{ki} < 2$, $\varepsilon_{ki} > 0$, $i^*i = 1$, $i_*i = 2$, $k = 1, 2, \dots, m$, i denotes the i -th aircraft, and note that $\left(-\frac{O_i^{(k)}}{R_{ki}} t^2 + 2O_i^{(k)} t - O_i^{(k)} R_{ki} \right) \Big|_{t=R_{ki}} = \left(-\frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right) \Big|_{t=R_{ki}} = 0$.

The sliding variable is defined as

$$s_{Gi} := s_{\sigma(t)i} \quad (7)$$

$$\text{where } s_{\sigma(t)i} := (s_{\sigma_1(t)i}^{(1)}, s_{\sigma_2(t)i}^{(2)}, \dots, s_{\sigma_m(t)i}^{(m)})^T.$$

The sliding mode is defined as

$$S_{Gi}: s_{\sigma(t)i} = 0 \quad (8)$$

The corresponding sliding manifold is

$$G_i: \{(\dot{x}_{1i}^T, \dot{x}_{2i}^T) \in \mathbb{R}^{2m}: s_{\sigma(t)i} = 0\} \quad (9)$$

The scheduling strategy is

if $t \in R_{ii}^{(k)}$ then $\sigma_{ki}(t) = i'$

where $R_{1i}^{(k)} := \{t: t \leq R_{ki}\}$, $R_{2i}^{(k)} := \{t: t > R_{ki}\}$, $i' = 1, 2$, and note that $s_{Gi}(0) = 0$.

Theorem 1: Selecting the MAPTFC (6) for the system (1), then all aerial vehicles achieve formation control at a prescribed time in the presence of disturbances.

Proof: For the control system, there is

$$\begin{aligned} \ddot{x}_{1i} &= \frac{\partial f_{1i}}{\partial x_{1i}} \dot{x}_{1i} + \frac{\partial f_{1i}}{\partial x_{2i}} \dot{x}_{2i} \\ &= \frac{\partial f_{1i}}{\partial x_{1i}} f_{1i} + \frac{\partial f_{1i}}{\partial x_{2i}} (f_{2i} + b_i u + d_i) \\ &= -w_{\sigma(t)i} + \frac{\partial f_{1i}}{\partial x_{2i}} d_i \end{aligned} \quad (10)$$

$$\begin{aligned} \ddot{x}_{ki}^{(1)} &= -\frac{o_i^{(k)} k_{ki}}{j_{ki}} \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)^{2-\frac{j_{ki}}{k_{ki}}} + \frac{2O_i^{(k)}}{R_{ki}} - \left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i \operatorname{sgn}(s_{i^*i}^{(k)}) - \\ &\quad \varepsilon_{ki} \operatorname{sgn}(s_{i^*i}^{(k)}) - \left[\frac{\partial f_{1i}}{\partial x_{2i}} d_i \right]_k \end{aligned} \quad (11)$$

$$\begin{aligned} s_{i^*i}^{(k)} &= o_i^{(k)} \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right) + \frac{j_{ki}}{k_{ki}} \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)^{\frac{j_{ki}}{k_{ki}}-1} \left(\ddot{x}_k^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} \right) \\ &= -\frac{j_{ki}}{k_{ki}} \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)^{\frac{j_{ki}}{k_{ki}}-1} \left(\left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i \operatorname{sgn}(s_{i^*i}^{(k)}) + \varepsilon_{ki} \operatorname{sgn}(s_{i^*i}^{(k)}) - \left[\frac{\partial f_{1i}}{\partial x_{2i}} d_i \right]_k \right) \end{aligned} \quad (12)$$

Consider the Lyapunov function

$$V_{i^*i}^{(k)} = \frac{1}{2} s_{i^*i}^{(k)2} \quad (13)$$

According to (12) and (13), there is

$$\begin{aligned} V_{i^*i}^{(k)} &= s_{i^*i}^{(k)} \dot{s}_{i^*i}^{(k)} \\ &= -\frac{j_{ki}}{k_{ki}} \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)^{\frac{j_{ki}}{k_{ki}}-1} \left(\left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i |s_{i^*i}^{(k)}| + \varepsilon_{ki} |s_{i^*i}^{(k)}| - \left[\frac{\partial f_{1i}}{\partial x_{2i}} d_i \right]_k s_{i^*i}^{(k)} \right) \\ &\leq -\frac{j_{ki}}{k_{ki}} \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)^{\frac{j_{ki}}{k_{ki}}-1} \varepsilon_k |s_{i^*i}^{(k)}| \\ &\leq 0 \end{aligned} \quad (14)$$

There is

$$\begin{aligned} \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)' &= -\frac{o_i^{(k)} q_k}{p_k} \left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)^{2-\frac{j_{ki}}{k_{ki}}} - \left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i \operatorname{sgn}(s_{i^*i}^{(k)}) \\ &\quad - \varepsilon_{ki} \operatorname{sgn}(s_{i^*i}^{(k)}) - \left[\frac{\partial f_{1i}}{\partial x_{2i}} d_i \right]_k \end{aligned} \quad (15)$$

In the case of $\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} = 0$ and $s_{i^*i}^{(k)} \neq 0$, since

$$\left(\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} \right)' \leq -\varepsilon_{ki} \operatorname{sgn}(s_{i^*i}^{(k)}) < 0 \quad (16)$$

$\dot{x}_{ki}^{(1)} - \frac{2O_i^{(k)}}{R_{ki}} t + 2O_i^{(k)} = 0$ is not an attractor. Therefore, according to (14) and (15), the sliding manifold $s_{i^*i}^{(k)} = 0$ is reached globally in finite time.

There is

$$\begin{aligned} \dot{x}_{ki}^{(1)} = & -\frac{o_i^{(k)} k_{ki}}{j_{ki}} \dot{x}_{ki}^{(1)2-\frac{j_{ki}}{k_{ki}}} - \left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i \operatorname{sgn}(s_{i^*i}^{(k)}) \\ & - \varepsilon_{ki} \operatorname{sgn}(s_{i^*i}^{(k)}) - \left[\frac{\partial f_{1i}}{\partial x_{2i}} d_i \right]_k \end{aligned} \quad (17)$$

$$\begin{aligned} s_{i^*i}^{(k)} = & o_i^{(k)} \dot{x}_{ki}^{(1)} + \frac{j_{ki}}{k_{ki}} \dot{x}_{ki}^{(1)\frac{j_{ki}}{k_{ki}}-1} \ddot{x}_k^{(1)} \\ = & -\frac{j_{ki}}{k_{ki}} \dot{x}_{ki}^{(1)\frac{j_{ki}}{k_{ki}}-1} \left(\left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i \operatorname{sgn}(s_{i^*i}^{(k)}) + \varepsilon_{ki} \operatorname{sgn}(s_{i^*i}^{(k)}) \right. \\ & \left. - \left[\frac{\partial f_{1i}}{\partial x_{2i}} d_i \right]_k \right) \end{aligned} \quad (18)$$

Consider the Lyapunov function

$$V_{i^*i}^{(k)} = \frac{1}{2} s_{i^*i}^{(k)2} \quad (19)$$

According to (18) and (19), there is

$$\begin{aligned} V_{i^*i}^{(k)} = & s_{i^*i}^{(k)} \dot{s}_{i^*i}^{(k)} \\ = & -\frac{j_{ki}}{k_{ki}} \dot{x}_{ki}^{(1)\frac{j_{ki}}{k_{ki}}-1} \left(\left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i |s_{i^*i}^{(k)}| + \varepsilon_{ki} |s_{i^*i}^{(k)}| \right) \\ & - \left[\frac{\partial f_{1i}}{\partial x_{2i}} d_i \right]_k s_{i^*i}^{(k)} \\ \leq & -\frac{j_{ki}}{k_{ki}} \dot{x}_{ki}^{(1)\frac{j_{ki}}{k_{ki}}-1} \varepsilon_k |s_{i^*i}^{(k)}| \\ \leq & 0 \end{aligned} \quad (20)$$

There is

$$\dot{x}_{ki}^{(1)} = -\frac{o_i^{(k)} q_k}{p_k} \dot{x}_{ki}^{(1)2-\frac{j_{ki}}{k_{ki}}} - \left\| \frac{\partial f_{1i}}{\partial x_{2i}} \right\| \bar{d}_i \operatorname{sgn}(s_{i^*i}^{(k)}) - \varepsilon_{ki} \operatorname{sgn}(s_{i^*i}^{(k)}) - \left[\frac{\partial f_{1i}}{\partial x_{2i}} d_i \right]_k \quad (21)$$

In the case of $\dot{x}_{ki}^{(1)} = 0$ and $s_{i^*i}^{(k)} \neq 0$, since

$$\dot{x}_{ki}^{(1)} \leq -\varepsilon_{ki} \operatorname{sgn}(s_{i^*i}^{(k)}) < 0 \quad (22)$$

$\dot{x}_{ki}^{(1)} = 0$ is not an attractor. Therefore, according to (20) and (21), the sliding manifold $s_{i^*i}^{(k)} = 0$ is reached globally in finite time.

Since $s_{Gi}(0) = 0$, and according to (14), (15), (20) and (21), the sliding manifold can be reached globally in finite time, $s_{Gi} \equiv 0$, which means that the global robustness to uncertainties is provided, and furthermore the state will converge to the equilibrium at the prescribed convergence time. Then there is $x_{ki}^{(1)} = 0$ and $\dot{x}_{ki}^{(1)} = 0$ for $t \geq t_{pi}^{(k)}$. It means that all aerial vehicles achieve formation control at a prescribed time in uncertainty. This completes the proof.

4. Conclusions

This paper presented an anti-disturbance prescribed time control strategy of multi-aircraft formation to meet the prescribed-time convergence performance requirement of cooperative errors. The multiple aircraft dynamics and kinematics model, as well as the formation error control model were developed. Based on this, a sliding mode control scheme with a prescribed time convergence was formulated to improve the convergence of multiple aircraft cooperative formation errors in the presence of uncertainties to satisfy the high-performance requirements of multi-aircraft cooperative control. In addition, a strict demonstration of multi-aircraft prescribed time formation based on Lyapunov stability theory was given, which guarantees that the cooperative formation errors can be converged at the prescribed time in the presence of disturbances and uncertainties. The suggested approach provides a viable solution to anti-disturbance cooperative formation control of multiple aircraft with prescribed time performance.

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