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Article

Analyzing the Development Trajectory of Sociomathematical Norms in a Ninth-Grade Classroom-A Case Study on the Lesson "Inscribed Angle"

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Abstract: This case study examines a standard ninth-grade lesson on "Inscribed Angle." Drawing on sociomathematical norm theory, the study analyzes classroom video recordings, transcripts, and reflective journals to investigate how teacher-student interactions shape mathematical understanding and facilitate the emergence of these norms. In a classroom environment that encouraged student expression, four types of sociomathematical norms were observed: (a) expressing personal ideas freely, (b) applying mathematical knowledge to problem-solving, (c) specifying mathematical elements, and (d) identifying special combinations of these elements. The teacher guided the development of these norms through discursive strategies such as verbal emphasis, strategic interruptions, and positive reinforcement. Methodologically, this study serves as an example of microgenetic analysis of classroom interactions, thereby enriching the theoretical dimensions of research on sociomathematical norms.

Keywords: sociomathematical norms; Inscribed Angle; classroom interaction; case study; microgenetic analysis

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1. Introduction

Since the 1990s, research in mathematics education has undergone a significant paradigm shift, moving from a predominantly cognitive perspective toward a sociocultural turn. This theoretical transformation recognizes that mathematical learning is not merely an individual cognitive process but fundamentally a social activity situated within specific cultural and classroom contexts. Against this backdrop, the conceptualization of norms governing mathematical discourse and practice has emerged as a critical area of inquiry.

The concept of sociomathematical norms was formally introduced by Yackel and Cobb in their seminal work on the development of mathematical reasoning in classroom settings. Unlike general social norms that regulate classroom behavior (such as taking turns or respecting others' opinions), sociomathematical norms specifically refer to the collectively established criteria for what constitutes an acceptable mathematical explanation, justification, and argumentation within a classroom community [1,2]. These norms shape students' understanding of what counts as mathematically different, sophisticated, or efficient, thereby fundamentally influencing their mathematical development.

The significance of sociomathematical norms lies in their role as the implicit framework that guides mathematical discourse. When students internalize these norms, they develop not only content knowledge but also mathematical practices and dispositions essential to meaningful participation in mathematical communities. As Stephan emphasizes, sociomathematical norms are collaboratively negotiated through ongoing classroom interactions, with teachers and students jointly constructing the standards for mathematical interpretation [2].

Existing research has examined sociomathematical norms through diverse methodological lenses. Conversation analysis has been employed to examine the micro-level interactions through which norms are negotiated and reinforced, while constant comparison methods have enabled researchers to identify patterns across different classroom settings [3,4]. Research perspectives have ranged from investigating specific teaching reform interventions to conducting cross-contextual comparative analyses that examine how norms manifest differently across various classroom microcultures [4,5]. These studies have collectively contributed to our understanding of what sociomathematical norms are and how they function in mathematics classrooms.

However, despite this growing body of literature, in-depth empirical studies on the developmental process of sociomathematical norms remain surprisingly scarce. While McClain and Cobb provided a foundational analysis of how sociomathematical norms developed in one first-grade classroom, similar longitudinal or microgenetic investigations in other grade levels and instructional contexts are largely absent from the literature [6]. This gap is particularly notable given the theoretical assertion that these norms emerge and evolve through specific interactional processes rather than being directly transmitted by teachers.

The developmental trajectory of sociomathematical norms warrants closer examination for several reasons. First, understanding how norms emerge in the initial stages of teacher-student interaction can illuminate the mechanisms through which mathematical classroom cultures are established. Second, tracing the evolution of these norms across different phases of instruction can reveal how teachers' discursive strategies shape students' mathematical dispositions. Third, identifying critical moments in norm development can inform teacher education and professional development programs, helping educators become more intentional in fostering productive mathematical discourse.

The present study addresses this research gap by examining a ninth-grade mathematics classroom where student expression was actively encouraged. Specifically, the study poses the following research question: How do sociomathematical norms emerge and develop in a ninth-grade mathematics classroom during the initial phase of teacher-student interaction?

2. Research Design

The case for this study was a ninth-grade lesson on "Inscribed Angle," drawn from a municipal-level "National Teacher Training Program" demonstration class in China. While serving as a teaching assistant for this program, the researcher observed various instructional approaches for the same topic and recorded the complete classroom process of the target lesson. Post-lesson interviews with the instructors and their explanations of the lesson design during the evaluation session confirmed that this was a routine lesson, representing a natural teaching scenario. Given the program's significant influence on teaching and research practices in China, this lesson can be considered reasonably representative.

Furthermore, the existing collaborative relationship between the researcher and the teacher facilitated access to authentic and valuable data within a high-trust environment. Moreover, because this lesson marked the first formal interaction between the teacher and students, it provided an ideal opportunity to observe the natural emergence of sociomathematical norms, thereby enhancing the validity of the findings.

In the analysis phase, the researcher repeatedly viewed the classroom video, produced detailed transcripts, and kept a reflective journal. Classroom interactions were then analyzed in depth, using the framework of sociomathematical norms to address the research questions.

3. Research Findings

3.1 The Sociomathematical Norm of "Encouraging Students to Express Themselves Freely"

At the start of the class, the teacher played a video of a soccer match and then invited students to share their thoughts.

As shown in Table 1, The teacher's opening question was open-ended, exemplifying the social norm of encouraging students to express their thoughts freely. At this initial stage, any response was acceptable, and the primary goal was to elicit student participation.

Table 1. Classroom teaching segment(1).

Turn	Speaker	Interaction
01	Teacher	Come on, come on! Let me invite a classmate to share. First, what did you see? Second, what are your thoughts?

3.2 The Sociomathematical Norm of "Using Mathematical Knowledge to Solve Problems"

The teacher then introduced a mathematical problem embedded in a real-world context.

As shown in Table 2, While the teacher's prompt appeared similar to the initial phase, asking for students' "feelings," the emphasis on "reading the question" and "how to solve it" signaled a shift. The expectation was no longer just general expression, but mathematically-grounded expression.

Table 2. Classroom teaching segment(2).

Turn	Speaker	Interaction
01	Teacher	Alright, let's hear from this female student. She just gave an excellent reading. Now, what did you feel after hearing it? How do you think we should address this issue?
02	Students	I think the two players should work together.
03	Teacher	Alright (no pause between "good" and the student's response), now look at this question. It shows our student didn't read the question carefully enough. So, what's my question? Yes, he's asking whether it's better to let you take the shot or to take it yourself. So, this question is what I'm asking. (Pause 1 second) Is it better for A to take the shot himself or for B to take it? That's the question, right? Alright, what do you think? (Pause 4 seconds)

The student's response, rooted in real-life experience ("players work together"), was mathematically irrelevant. The teacher's interruption communicated that such experience-based reasoning was not an acceptable mathematical explanation, thereby beginning to establish a new norm.

As shown in Table 3, The teacher's repeated use of phrases like "this problem" and "so" served as verbal emphasis, guiding students to articulate the "mathematical problem" through a fill-in-the-blank technique. This interaction pulled the entire class toward the emerging sociomathematical norm of using mathematical knowledge to solve problems.

Table 3. Classroom teaching segment(3).

Turn	Speaker	Interaction
01	Teacher	When we're stuck on a problem, we should break it down into something we've already studied. Hey, what do you guys think? A, B, who's better at soccer? How should we rephrase this question?
02	Students	Math question. (About five students respond in unison)

As shown in Table 4, Even though "shortest path" was not directly applicable to the target lesson, the teacher acknowledged and emphasized the student's contribution. This acceptance reinforced the norm that mathematical knowledge-even if not perfectly aligned-was valued, encouraging further mathematical participation.

Table 4. Classroom teaching segment(4).

Turn	Speaker	Interaction
01	Teacher	Oh, what kind of math problem? Come on, who's up for it?
02	Students	The problem of finding the shortest path.
03	Teacher	Alright, the first shortest path-whatever method you use, isn't it the shortest path? Great, so what if there's another one?

3.3. The Sociomathematical Norm of "Concretizing Mathematical Elements"

The teacher then probed how to solve the mathematical problem.

As shown in Table 5, When a student mentioned the key word "angle," the teacher interrupted, prompting them to specify which angle. This interaction exemplifies the emerging sociomathematical norm of "concretizing mathematical elements"-moving from vague references to precise mathematical language.

Table 5. Classroom teaching segment(5).

Turn	Speaker	Interaction
01	Teacher	Okay, come on, tell me. What do you mean?
02	Students	Well, he plays soccer from a certain angle.
03	Teacher	Oh, what kind of angle?

As shown in Table 6, After students identified two angles, the teacher interrupted again, guiding them not only to specify the angles but also to clarify the relationship between them. This further deepened the norm of concretizing mathematical elements.

Table 6. Classroom teaching segment(6).

Turn	Speaker	Interaction
01	Students	It's about connecting AN and AM.
02	Teacher	Good.
03	Students	Also BA, BM.
04	Teacher	In other words, there's an angle (0.5-second pause), and I just need to compare what's there. What's the difference between angle A and angle B? (Teacher facing the class)
05	Students	Size relationship. (Two students respond)

After transforming the real-world problem into a mathematical one, the teacher directed students' attention to a diagram and reviewed previously learned concepts related to comparing angles.

As shown in Table 7, Through verbal emphasis, the teacher highlighted all relevant mathematical elements, affirming even those that were not the immediate target (e.g., radius) while steering attention toward the most pertinent ones (e.g., arc). This reinforcement solidified the norm of concretizing mathematical elements.

Table 7. Classroom teaching segment(7).

Turn	Speaker	Interaction
01	Teacher	Oh, that's great! What kind of angle did we just learn about?
02	Students	Central angle. (Answer in unison)

Turn	Speaker	Interaction
03	Teacher	Well, excellent! This student is truly outstanding. He actually grasped the essence-what exactly is the relationship between the central angle of a circle and the angle?
04	Students	Together. (In unison)
05	Teacher	Look, it's familiar-the central angle. Everyone found it, right? So, apart from circles and angles in the diagram, are there any other familiar elements? Alright, let me give you a quick reminder of some elements we just learned. Now, it's your turn to share.
06	Students	Radius.
07	Teacher	The concept of radius isn't something we've just learned-it's actually taught in elementary school. So, is there anything else we should know about circles?
08	Students	Arc.
09	Teacher	Oh, what else?
10	Students	Arc. (In unison)
11	Teacher	Oh, and the arc-excellent. So what exactly is this arc? It's a special combination of a circle and an angle.
12	Students	Arc.
13	Teacher	Right? So, what does the central angle O correspond to?
14	Students	BC arc. (Students answer together)

3.4. The Sociomathematical Norm of "Exploring Special Combinations of Elements"

As shown in Table 8, Both student responses focused on general mathematical properties (angle equality and same arc), which did not fully align with the lesson's targeted norm of identifying special cases of element combinations. Notably, the teacher did not reject these answers but used them as a springboard for further guidance.

Table 8. Classroom teaching segment(8).

Turn	Speaker	Interaction
01	Teacher	Okay, come on, tell me why you think it's special. Explain your reasons.
02	Students	Well, I measured with a protractor and found all three angles were equal.
03	Teacher	Well, come on, tell me why you chose these three shapes instead of mine. What makes them special? (Pause 5 seconds) Why did you draw them this way? Is it because you had no other ideas? Right? Now, what about the other students? Any thoughts? Maybe draw something unique, or something special. Wait, what made the previous ones special? The vertices-where do they go from the center? The angles' vertices-where do they go from the center?
04	Students	The circumference.
05	Teacher:	Isn't that really special, isn't it?
06	Teacher	So here he's drawing. During the process, we need to think carefully. It's not just random, right? Hey, do you think there's something special about what he's drawing? Wait, does this student have any thoughts?
07	Students	They are all drawn on the same arc.
08	Teacher	Oh (following the student's answer), first of all, are they all on the same arc, right? Very good.

Ten minutes earlier, the teacher had conducted a fill-in-the-blank activity with the entire class, and all students responded correctly.

As shown in Table 9, This interaction established the foundation for exploring special cases by contrasting the familiar (central angle) with the novel (inscribed angle).

Table 9. Classroom teaching segment(9).

Turn	Speaker	Interaction
01	Teacher	They're superimposed on top of each other. Does this superposition have any distinctive features? Is it about the-? Where is the vertex of this angle placed?
02	Students	On the center. (Students answer together)
03	Teacher	Great! Today we'll explore another type of angle related to angles and circles. If we keep combining angles and circles, where do you think the vertex should be placed to highlight this unique feature?
04	Students	On the circle. (Synchronized response)

To better illustrate the developmental trajectory of sociomathematical norms in this ninth-grade "Inscribed Angle" lesson, the normative development stages, core norms, and key interactive behaviors are summarized in Table 10.

Table 10. Developmental Trajectory of Sociomathematical Norms.

Developmental Stage	Core Sociomathematical Norm	Key Interaction	Defining Feature
Initial Phase	Encouraging student expression	Teacher poses open-ended question (e.g., "What do you think about the video?") and invites students to share.	Open-ended; emphasizes sharing personal ideas.
Transition to Mathematization	Using mathematical knowledge to solve problems	Teacher interrupts nonmathematical responses (e.g., "players work together") and guides students to articulate "mathematical problems" through fill-in-the-blank questions, emphasizing the problem's core.	Rejects real-life reasoning; focuses on mathematizing the problem.
Deepening Analysis	Concretizing mathematical elements	Teacher asks "What kind of angle?" and "What relationship between angles?" with emphasis on mathematical elements (e.g., "inscribed angle," "arc"), while acknowledging non-target but relevant answers (e.g., "shortest path").	Specifies mathematical language; clarifies elements.
Advanced Exploration	Identifying special combinations of elements	Teacher guides thinking from "vertex on the center" to "vertex on the circumference," probing the "special reason" for drawing angles and affirming discovery of "angles on the same arc."	Focuses on element relationships; emphasizes identifying special cases.

4. Conclusion and Discussion

This study reveals that teachers facilitate students' construction and negotiation of sociomathematical norms through classroom interactions, employing discursive techniques such as verbal emphasis, strategic interruptions, and positive reinforcement. When student responses deviated from the targeted norms, the teacher provided guidance through followup questions or subtle corrections. Even when responses aligned with general norms but fell short of specific lesson requirements, the teacher offered constructive feedback and further direction.

The research demonstrates that sociomathematical norms are not directly imparted by teachers but rather develop gradually through collective and individual interactions between teachers and students. From a temporal perspective, students may encounter adaptation challenges when transitioning between different interaction patterns, highlighting the importance of explicit and implicit scaffolding.

This study offers two main academic contributions. Methodologically, it exemplifies a microgenetic approach to analyzing classroom interactions, revealing how the teacher's discursive moves shape the development of sociomathematical norms. This provides a replicable analytical framework for classroom research. Theoretically, it enriches the existing literature by illustrating the process through which moment-by-moment interactions guide the emergence and consolidation of these norms, thereby expanding the analytical dimensions of related studies.

However, this study has certain limitations. First, it was based on a single lesson case, and the generalizability of the findings requires further validation across different instructional contexts. Second, the preexisting relationship between the researcher and the teacher, while facilitating access, may have introduced potential bias in data interpretation. Third, the study focused exclusively on teacher-student verbal interactions, potentially overlooking nonverbal cues and student-student dynamics that may also contribute to norm development.

Future research could address these limitations by collecting data from a broader range of classrooms, incorporating different educational stages and cultural backgrounds. Additionally, longitudinal studies could examine how sociomathematical norms evolve over extended periods, and comparative studies could explore how different instructional approaches influence norm development. Such investigations would further deepen our understanding of the complex processes through which mathematical communities of practice are established in classroom settings.

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