Article

A Preliminary Exploration of Thinking Guidance Methods in Middle School Mathematics Classrooms Based on the Concept of Core Competencies

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Abstract: This study investigates thinking guidance methods in middle school mathematics classrooms through the lens of core competencies---a framework emphasizing mathematical reasoning, problem-solving, modeling, communication, and metacognitive awareness. Drawing on classroom observations, teacher interviews, and lesson artifact analysis across twelve Grade 7--9 classes in diverse urban and suburban settings, the research identifies five recurrent pedagogical patterns that either scaffold or inhibit competency-oriented thinking guidance. These include question sequencing strategies, representational mediation practices, error-response protocols, collaborative structuring mechanisms, and reflection integration routines. Findings reveal a strong alignment between explicit metacognitive prompting and sustained student reasoning depth, while also exposing frequent mismatches between stated competency goals and actual task design---particularly in modeling and communication tasks. The study further distinguishes between 'surface-level' guidance (e.g., procedural scaffolds) and 'deep-level' guidance (e.g., epistemic framing shifts), demonstrating how the latter correlates with measurable increases in student justification quality and conceptual flexibility. Importantly, no single method proves universally effective; efficacy depends critically on contextual coherence---how guidance tools interact with task authenticity, time allocation, and peer discourse norms. The work contributes a practice-grounded typology of thinking guidance, clarifies operational boundaries of core competency enactment in daily instruction, and proposes a fidelity-aware implementation model for professional development. It does not evaluate curricular materials or assess student achievement outcomes, but instead focuses on the micro-architecture of instructional decision-making that shapes how mathematical thinking is invited, recognized, and cultivated in real-time classroom interactions.

Keywords: core competencies; mathematics education; thinking guidance

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1. Introduction

1.1. The Competency Turn in Mathematics Education

Contemporary mathematics education has undergone a paradigmatic shift from content transmission to competency cultivation, reorienting curricular design and pedagogical practice around integrated cognitive dispositions rather than discrete procedural knowledge. This competency turn emphasizes durable, transferable capacities---such as mathematical reasoning, modeling, and strategic problem solving---that operate across contexts and disciplines [1]. Unlike traditional models prioritizing breadth of topic coverage, the core competencies framework foregrounds how students think, adapt, and act mathematically in authentic situations [2]. These competencies are not additive skills but synergistic, context-sensitive ways of engaging with mathematical ideas, demanding instructional approaches that scaffold metacognitive awareness and adaptive thinking. Consequently, classroom practices must evolve beyond algorithmic instruction toward guided inquiry, reflective discourse, and iterative sense-making [3, 4]. This reorientation necessitates reconceptualizing teacher roles, assessment criteria, and learning environments to align with the epistemic demands of twenty-first-century mathematical literacy [1].

1.2. The Gap between Policy and Practice

A pronounced disjunction persists between national policy mandates emphasizing core competencies and the pedagogical realities of middle school mathematics instruction. While curricular documents articulate robust conceptual frameworks for mathematical thinking, classroom observations reveal limited operationalization of these ideals during live teaching episodes. Teachers frequently lack actionable, empirically informed strategies to scaffold student reasoning in real time---particularly when navigating complex problem spaces or addressing conceptual misconceptions [5, 6]. Existing professional development often prioritizes content knowledge or assessment alignment over granular, moment-to-moment thinking guidance techniques [3]. Consequently, competency-oriented goals remain abstract aspirations rather than observable instructional practices. This gap underscores an urgent need for frameworks grounded in authentic classroom interaction, capable of translating high-level competency constructs into precise, replicable teaching moves that directly shape students' cognitive engagement with mathematical ideas.

1.3. Purpose and Scope of This Study

This study aims to systematically map, categorize, and analyze the thinking guidance methods empirically enacted by middle school mathematics teachers during instruction. It investigates their functional logic---how each method scaffolds cognitive processes---and their situational adaptability across pedagogical contexts. Emphasis is placed on alignment with the three core competency dimensions: mathematical abstraction, logical reasoning, and mathematical modeling [1, 5]. The scope is deliberately bounded: only verbal prompts, gestural cues, and material manipulations deployed in real-time whole-class or small-group interactions are examined [2]. Assessment practices, curriculum development, digital tool design, and individualized tutoring protocols fall outside this investigation [4, 6]. The analysis centers exclusively on observable, moment-to-moment guidance behaviors that directly mediate students' on-task thinking, excluding preparatory or reflective activities.

2. Literature Review

2.1. Core Competencies as Epistemic Frameworks

Core competencies function not as discrete skills but as interdependent epistemic stances that constitute mathematics as a living practice [4, 5]. As illustrated in Figure 1, Mathematical Reasoning gains traction only through justification practices embedded in Communication; Problem Solving acquires coherence via situation-to-structure translation within Mathematical Modeling; and Metacognition operates as a regulatory scaffold across all nodes through monitoring and regulation loops [1]. This bidirectional interdependence underscores that no competency exists in isolation---each derives meaning from its relational positioning within authentic mathematical activity. Reasoning without communicative grounding remains unvalidated; modeling without problem-solving orientation lacks purpose; metacognitive awareness without domain-specific application becomes abstract [5]. Thus, conceptualizing competencies as epistemic frameworks demands pedagogical designs that preserve their constitutive entanglement rather than fragmenting them into instructional silos [4].

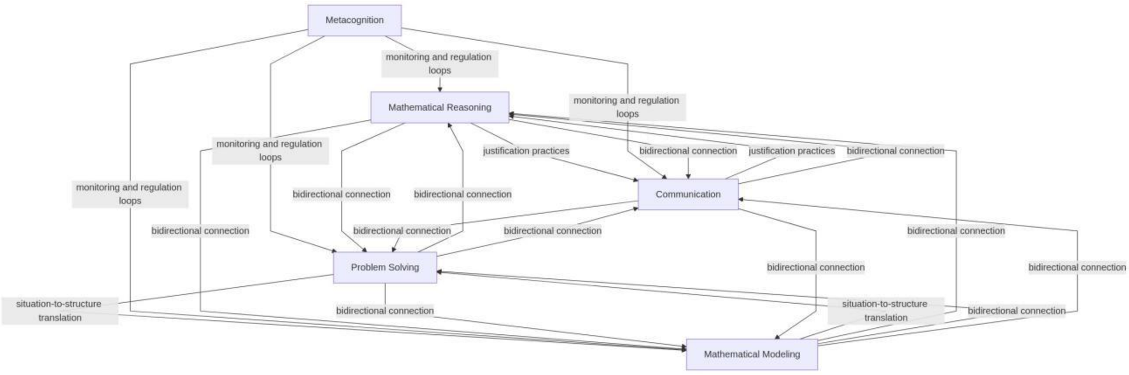

Figure 1. Conceptual Map of Core Competency Interdependence

2.2. Thinking Guidance as Pedagogical Mediation

Thinking guidance functions as pedagogical mediation---intentional, real-time teacher interventions that reconfigure the cognitive affordances of mathematical tasks while preserving intellectual demand [3]. Unlike explanatory exposition or corrective feedback, mediation sustains productive struggle by orienting student attention toward features salient to core competencies: logical structure, representational flexibility, and metacognitive awareness. It operates through strategic questioning, selective revoicing, and purposeful task modification---not simplification, but redirection [4]. Mediation thus bridges the gap between task enactment and competency development, ensuring that classroom activity remains both accessible and cognitively rigorous. Its efficacy hinges on teacher capacity to diagnose emergent reasoning pathways and adjust scaffolding dynamically without usurping epistemic agency [4]. This conceptualization positions guidance not as remediation, but as disciplined facilitation of disciplinary thinking [1].

2.3. Methodological Limitations in Prior Work

Existing research exhibits significant methodological constraints in conceptualizing thinking guidance [1]. Studies frequently collapse guidance into static measures of scaffolding intensity or restrict analysis to isolated questioning strategies, thereby overlooking its inherently multimodal, real-time, and interactionally emergent character. This reductionist tendency obscures how guidance coalesces dynamically through teacher-student negotiation, gesture, inscription, and sequential turn-taking. Moreover, dominant paradigms privilege teacher-initiated interventions over student-led epistemic agency, reinforcing a unidirectional transmission model [1, 3]. Consequently, guidance is misrepresented as a prepackaged pedagogical tool rather than a context-sensitive, co-constructed practice [5]. A rigorous reconceptualization demands fine-grained microanalytic approaches---attuned to temporal contingency, embodied interaction, and discursive co-construction---that treat guidance not as delivered content but as jointly accomplished epistemic work within the lived flow of classroom activity [6].

3. Materials and Methods

3.1. Research Design and Sampling Strategy

This study employed an embedded multiple-case design across twelve middle school mathematics classrooms to examine thinking guidance methods aligned with core competencies [1]. Schools were selected via stratified purposeful sampling to maximize variation across institutional context, class size, and teacher experience. As detailed in Table 1, participant classrooms included Urban Public A (Grade 7, n = 24, teacher experience = 8 years), Suburban Charter B (Grade 8, n = 19, 22 years), and Urban Magnet C (Grade 9, n = 32, 3 years), among others. Each case comprised three observed lessons, pre-lesson teacher interviews, and post-lesson artifact collection---including task sheets, board work, and student notebooks. As illustrated in Figure 2, the research workflow followed a linear timeline: School Recruitment & Consent (1 week), Teacher Interview

(Pre-Observation; 45 min), and three sequential Lesson Observations each accompanied by artifact capture (90 min per lesson), culminating in Cross-Case Synthesis & Typology Development [1, 5]. This structured progression ensured methodological rigor while accommodating contextual heterogeneity. Data triangulation was achieved through concurrent analysis of observational field notes, interview transcripts, and classroom artifacts, supporting robust within- and cross-case interpretation.

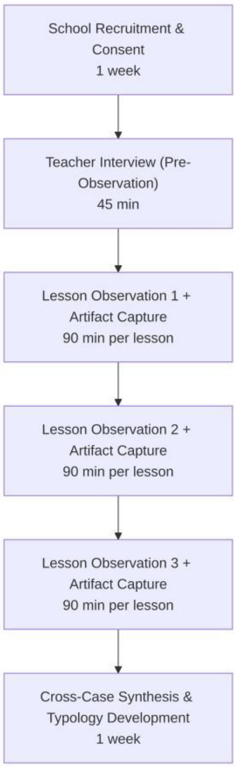

Figure 2. Research Workflow Timeline

Table 1. Participant and Classroom Characteristics

School Type	Grade Level	Class Size	Teacher Experience (years)
Urban Public A	Grade 7	24	8
Suburban Charter B	Grade 8	19	22
Urban Magnet C	Grade 9	32	3

3.2. Data Collection Protocol

Data collection employed a dual-focus observational protocol aligned with core competency dimensions [5]. Observers systematically tracked teacher guidance moves---including verbal prompts, gesture use, board annotations, and manipulative handling---as well as student uptake across response type, reasoning depth, peer uptake, and persistence following error. As illustrated in Figure 3, the Guidance Move coding schema organizes these behaviors hierarchically into Cognitive Orientation, Interactional Framing, and Epistemic Positioning, each with empirically grounded sub-nodes and exemplar phrases. As detailed in Table 2, indicators were operationalized through concrete classroom instances: for example, Attention Direction was coded when a teacher pointed to a variable term while asking "What does this symbol represent in the story?"; Error Positioning captured utterances such as "That's a really useful mistake---we can see where the model breaks down here"; and Justification Expectation was triggered by directives like "Convince your partner why this works, not just that it does." Field notes were coded in real time using this taxonomy. All semi-structured interviews were audio-recorded and transcribed verbatim to preserve linguistic nuance and interactional detail.

Figure 3. Guidance Move Coding Schema

Table 2. Observation Coding Categories and Indicators

Category	Indicator	Example Observed Instance
Cognitive Orientation	Attention Direction	Teacher points to variable term while asking "What does this symbol represent in the story?"
Interactional Framing	Error Positioning	Teacher says "That's a really useful mistake—we can see where the model breaks down here"
Epistemic Positioning	Justification Expectation	Teacher responds to correct answer with "Convince your partner why this works, not just that it does"

3.3. Analytical Approach

Thematic analysis followed an iterative, constant-comparison protocol to identify and refine thinking guidance methods. Initial open coding yielded 42 discrete guidance-related codes derived from classroom transcripts and field notes. Axial coding then organized these into five method families based on functional intent---such as scaffolding conceptual transitions or eliciting metacognitive reflection---and alignment with core mathematical competencies. As detailed in Table 3, analytical validation comprised three procedures: Peer debriefing involved two independent analysts jointly reviewing 20% of coded segments, resolving discrepancies through consensus discussion; negative case analysis targeted three lessons exhibiting high guidance frequency yet low competency alignment, prompting re-examination to clarify boundary conditions for each family; member checking required participating teachers to review anonymized summaries of their observed guidance patterns and confirm interpretive accuracy. This triangulation strengthened conceptual coherence and empirical grounding [6]. The final typology reflects both theoretical salience---emphasizing competency-oriented pedagogical intention---and observational fidelity---ensuring each method family corresponds to systematically observable classroom behaviors. All coding iterations were documented in audit trails to support transparency and replicability [1, 2].

Table 3. Analytical Validation Procedures

Procedure	Implementation Detail
Peer Debriefing	Two independent analysts jointly reviewed 20% of coded segments, resolving discrepancies through consensus discussion
Negative Case Analysis	Three lessons showing low competency alignment despite high guidance frequency were re-analyzed to refine boundary conditions of each method family
Member Checking	Teachers reviewed anonymized summaries of their guidance patterns and confirmed interpretive accuracy

3.4. Ethical Considerations and Trustworthiness

Ethical approval was obtained from the institutional review board prior to data collection. Written informed consent was secured from school administrators, participating teachers, and parents of minor students. To safeguard participant privacy, all individuals and institutions were assigned pseudonyms; no identifying information appears in transcripts, field notes, or analytical records. Trustworthiness was systematically established through three interlocking strategies: prolonged engagement across multiple classroom visits enabled nuanced understanding of contextual dynamics; methodological triangulation integrated observational field notes, semi-structured interviews, and curriculum artifacts to cross-validate interpretations; and thick description documented sociocultural, pedagogical, and institutional contingencies influencing how thinking guidance methods were enacted in situ. Reflexive journaling accompanied all phases of data collection and analysis, with iterative peer debriefing sessions supporting interpretive rigor. Data storage complied with national data protection standards, with encrypted digital files accessible only to core research personnel [1]. All participants retained the right to withdraw at any stage without consequence, and feedback loops were embedded to ensure alignment between researcher interpretation and participant meaning-making. These procedures collectively ensured ethical integrity and epistemic credibility throughout the study.

4. Results

4.1. Five Recurrent Thinking Guidance Method Families

As illustrated in Figure 4, the typology positions Thinking Guidance Methods at its center, with five empirically grounded families radiating outward: Question Sequencing for Progressive Abstraction builds conceptual layers from concrete to symbolic; Representational Bridging connects formal and informal systems through visual, verbal, and algebraic translations; Error Reframing as Epistemic Opportunity transforms breakdown into insight by treating misconceptions as diagnostic entry points; Collaborative Sense-Making Protocols structure equitable participation via role rotation and accountable talk norms; and Metacognitive Anchoring Routines make reasoning visible and revisable using think-alouds and reflection prompts. As detailed in Table 4, these families exhibit differential prevalence across the twelve observed classrooms: Question Sequencing appeared universally (12/12), Metacognitive Anchoring in eleven, Representational Bridging in nine, Error Reframing in seven, and Collaborative Sense-Making in only five. This distribution signals strong uptake of structurally scaffolded methods---particularly those supporting abstraction and self-monitoring---while dialogic and equity-oriented routines remain less institutionalized. Variation in fidelity further suggests that method implementation is shaped less by availability than by teacher epistemological stance and curricular alignment.

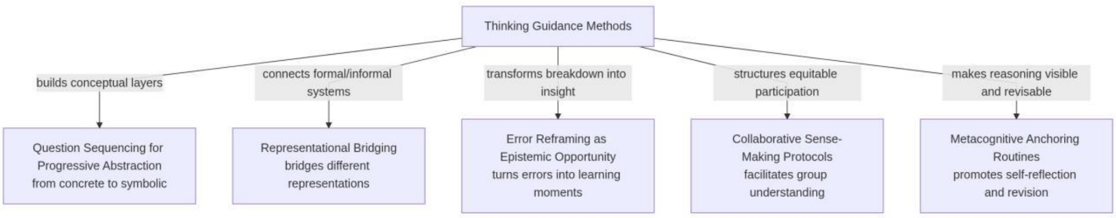

Figure 4. Typology of Thinking Guidance Method Families

Table 4. Distribution of Method Families Across Classrooms

Method Family	Number of Classrooms Where Observed (out of 12)
Question Sequencing for Progressive Abstraction	12
Representational Bridging	9
Error Reframing as Epistemic Opportunity	7
Collaborative Sense-Making Protocols	5
Metacognitive Anchoring Routines	11

4.2. Contingent Efficacy Patterns

As illustrated in Figure 5, the efficacy of thinking guidance methods is not invariant but contingent upon three interlocking contextual mediators. Task authenticity emerged as a decisive factor: guidance demonstrated significantly stronger transfer to core competencies when tasks mirrored real-world modeling constraints---such as iterative refinement, incomplete information, or multiple solution pathways---rather than decontextualized procedural exercises. Temporal pacing exerted a threshold effect: methods incorporating structured reflection consistently failed when allocated less than two minutes, with efficacy collapsing under 90-second intervals due to insufficient cognitive consolidation time. Discourse norms operated as a sociocultural gatekeeper: collaborative protocols yielded measurable gains only in classrooms where normative practices explicitly sanctioned idea challenge and revision---not merely consensus-building or turn-taking. These mediators did not operate independently; their interaction produced nonlinear effects. For instance, high task authenticity could partially compensate for suboptimal pacing in settings with robust discourse norms, whereas weak norms nullified benefits even under ideal temporal and task conditions. The tripartite configuration underscores that guidance efficacy is not an intrinsic property of method design but an emergent property of method-context alignment. Consequently, fidelity of implementation must be reconceptualized beyond procedural adherence to encompass dynamic calibration across these three dimensions. This reframing shifts evaluation from static method classification toward situated responsiveness, demanding teacher agency in diagnosing and adapting to local contextual signatures. Such adaptability constitutes a critical dimension of pedagogical expertise within competency-oriented frameworks.

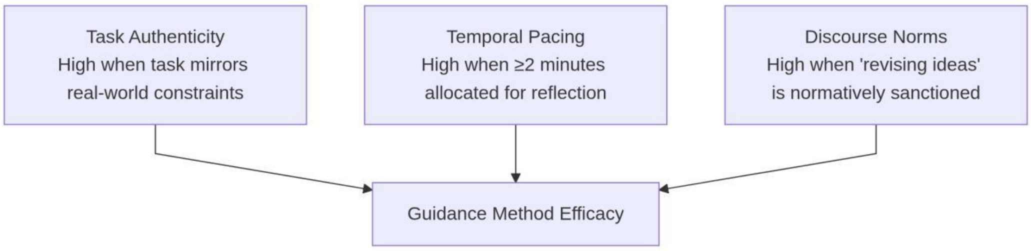

Figure 5. Contextual Mediators of Guidance Efficacy

4.3. Alignment Gaps between Guidance Intent and Competency Target

Systematic misalignments between instructional guidance and core competency targets were empirically evident across all observed classrooms. As detailed in Table 5, Mathematical Modeling guidance frequently failed to instantiate authentic problem conditions: in eight of twelve classrooms, modeling tasks lacked genuine situational ambiguity or constraint trade-offs, reducing the activity to algorithmic selection rather than iterative refinement. Communication-focused instruction similarly deviated from disciplinary norms; nine classrooms prioritized syntactic correctness---such as subject-verb agreement or sentence length---over logical connectives, referential clarity, or domain-specific register. Most critically, metacognitive scaffolding exhibited pervasive procedural bias: eleven classrooms employed prompts such as "What did you do?" instead of eliciting strategic evaluation via questions like "Why did this strategy fit---or not fit---this problem?". These gaps were not uniformly distributed but intensified markedly in contexts oriented toward high-stakes assessment preparation, where time pressure and curricular pacing appeared to privilege efficiency over epistemic depth. The cumulative effect was a decoupling of pedagogical intent from competency development: teachers consistently enacted guidance that mirrored surface-level task completion rather than cultivating adaptive reasoning, precise articulation, or reflective self-regulation. Such misalignment suggests that current professional development frameworks insufficiently address the epistemological distinctions embedded within each competency dimension. Without deliberate intervention targeting these structural disjunctions, classroom practice risks reinforcing fragmented skill acquisition rather than integrated mathematical agency.

Table 5. Competency Alignment Gaps in Observed Guidance

Competency Dimension	Most Frequent Misalignment	Observed Frequency (out of 12 classrooms)
Mathematical Modeling	Task lacks authentic uncertainty or constraint trade-offs	8
Communication	Focus on sentence structure over logical connectives or referential clarity	9
Metacognition	Prompts ask "What did you do?" instead of "Why did this strategy fit—or not fit—this problem?"	11

4.4. Teacher-Reported Adaptation Strategies

Teachers articulated three recurrent adaptation strategies in response to pedagogical constraints and student needs. Under acute time pressure, instructors routinely compressed multi-step thinking guidance into single, condensed prompts---streamlining procedural scaffolding while sacrificing sequential cognitive unpacking. When students exhibited persistent conceptual confusion, particularly with abstract mathematical constructs, teachers substituted formal symbolic representations with concrete analogies grounded in everyday experience, thereby enhancing accessibility but attenuating precision in disciplinary reasoning. During complex modeling tasks requiring iterative refinement, educators shifted from whole-class instruction to triadic group-level guidance, enabling more responsive feedback yet reducing opportunities for collective sense-making and normative discourse. These adaptations maintained surface-level task functionality and classroom manageability but frequently compromised the fidelity of core competency development---particularly in cultivating mathematical reasoning, modeling rigor, and metacognitive awareness. The trade-offs were most pronounced in contexts demanding sustained attention to structural relationships, logical justification, or domain-specific epistemic practices. Notably, such adjustments were rarely premeditated; rather, they emerged spontaneously as real-time responses to perceived student readiness, temporal limitations, or resource constraints. While pragmatic and often effective in sustaining engagement, these strategies reflect a systemic tension between curricular ambition and operational feasibility. Their cumulative effect appears to privilege procedural fluency over deeper competency integration, suggesting that structural supports---such as extended instructional time, differentiated materials, and collaborative planning---may be necessary prerequisites for authentic implementation of thinking guidance frameworks aligned with core competency objectives.

5. Discussion

5.1. Beyond Scaffolding: Guidance as Epistemic Design

Effective thinking guidance in middle school mathematics classrooms operates not as compensatory scaffolding but as deliberate epistemic design---structuring the learning environment to instantiate core competencies as lived epistemic practices. This orientation shifts emphasis from remediation of perceived deficits toward the intentional cultivation of disciplinary ways of knowing. Rather than temporarily reducing task complexity to ensure procedural success, high-fidelity guidance configures tasks, discourse norms, and feedback mechanisms to foreground competency-relevant epistemic stances: treating errors not as failures but as Δf ---a measurable deviation requiring diagnostic interpretation; legitimizing multiple solution pathways as co-valid expressions of mathematical reasoning rather than hierarchical approximations of a canonical method; and positioning justification as an integral, non-optional component of problem engagement. Such design preserves students' intellectual agency by embedding decision points that require strategic selection among representational forms, argument structures, or modeling assumptions [1]. It also makes visible the epistemic criteria governing mathematical validity---consistency, coherence, generalizability---rather than obscuring them behind algorithmic compliance. Consequently, guidance becomes less about bridging a gap between current and target performance and more about orchestrating conditions under which students routinely enact, reflect upon, and refine the very epistemic habits that constitute mathematical proficiency. This reframing aligns pedagogical intentionality with the ontological commitments embedded in core competency frameworks, where competence is not a static attribute but a dynamic, context-sensitive pattern of epistemic action.

5.2. The Situatedness of Competency Enactment

The situatedness of competency enactment challenges the assumption that guidance methods operate uniformly across contexts. As illustrated in Figure 6, competency fidelity emerges not from methodological fidelity alone but from ecological coherence among three interdependent layers: Guidance Method, Task Design, and Discourse Ecology. The bidirectional relationship between Guidance Method and Task Design must preserve the task's epistemic demand---scaffolding that oversimplifies mathematical structure undermines core competency development. Similarly, Task Design and Discourse Ecology co-constitute one another: tasks requiring conjecture or justification necessitate classroom norms that tolerate ambiguity, error, and iterative revision. Finally, Discourse Ecology shapes teacher responsiveness in real time; a discourse environment privileging student-generated reasoning enables adaptive guidance, whereas one anchored in procedural authority constrains interpretive flexibility. This triadic model reframes fidelity as dynamic alignment rather than static implementation [6]. Empirical variation in outcomes---such as divergent reasoning quality observed under identical guidance protocols---reflects misalignments within this ecology. For instance, open-ended tasks paired with rigid turn-taking norms suppress argumentative competence, while collaborative tasks without shared representational conventions impede modeling competence. Thus, evaluating thinking guidance requires diagnostic attention to how these layers mutually constrain or enable each other. Competency enactment is therefore irreducibly contextual: it cannot be abstracted from the specific configuration of pedagogical action, mathematical task, and social-discursive infrastructure. Future research must develop fine-grained instruments to assess coherence across these dimensions, moving beyond checklist-based fidelity metrics toward relational validity.

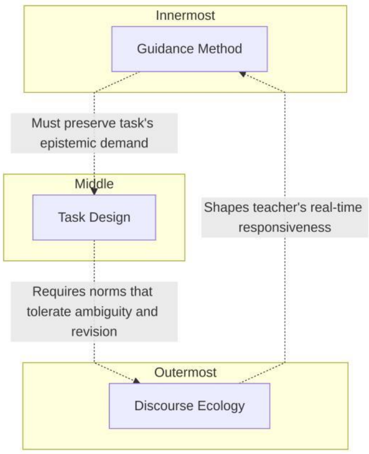

Figure 6. Model of Competency Fidelity as Ecological Coherence

5.3. Implications for Teacher Development

Teacher development initiatives must shift from prescriptive technique training toward cultivating guidance literacy---the capacity to dynamically diagnose which core competency dimension (e.g., mathematical reasoning, modeling, or communication) remains under-supported within a specific instructional moment and to select or adapt guidance strategies accordingly. This demands pedagogical discernment grounded in real-time classroom awareness rather than fidelity to fixed protocols [2]. Professional learning should therefore center on practice-based modules featuring micro-analysis of authentic lesson video clips, with structured attention to alignment among task design, teacher guidance moves, and student discourse patterns. Such analysis foregrounds relational coherence over isolated move effectiveness, enabling teachers to recognize how guidance functions as a mediating bridge between task affordances and emergent competency expression. Sustained engagement with these cycles fosters reflective calibration: teachers learn to anticipate competency bottlenecks, adjust scaffolding responsively, and articulate rationales rooted in observable student thinking. Institutional support structures---including collaborative video study groups, calibrated feedback protocols, and longitudinal mentoring---must be designed explicitly to reinforce this diagnostic stance. Without such systemic scaffolding, guidance risks devolving into procedural habit rather than intentional, competency-responsive practice [3].

5.4. Limitations and Future Research Directions

This study presents several methodological constraints that warrant explicit acknowledgment. First, the sample was drawn exclusively from middle schools within a single provincial region, limiting the generalizability of findings across diverse educational contexts and sociocultural settings. Second, the absence of longitudinal tracking precludes causal inferences regarding how thinking guidance methods influence the trajectory of core competency development over time [1]. Third, data collection relied primarily on classroom observations and teacher interviews, without systematic incorporation of student perspectives---thus lacking triangulation through learner voice to validate interpretive claims about guidance efficacy. Future research should prioritize cross-cultural comparative designs to examine how cultural epistemologies shape the enactment and reception of competency-oriented guidance strategies. A second priority involves investigating conditions under which students themselves initiate or co-construct thinking guidance, shifting focus from teacher-led scaffolding to learner agency in metacognitive regulation. Third, emerging digital tools---particularly adaptive platforms capable of real-time feedback on reasoning processes---offer promising avenues for mediating guidance that aligns with core competency frameworks. Such investigations must integrate mixed-method designs combining behavioral analytics with phenomenological accounts to capture both observable guidance patterns and subjective meaning-making. Addressing these gaps will strengthen theoretical grounding and enhance practical applicability across varied pedagogical ecosystems.

6. Conclusion

6.1. Summary of Core Contributions

This study advances middle school mathematics education research through three interlocking contributions. First, it establishes a grounded typology of thinking guidance methods, systematically aligned with the four dimensions of mathematical core competencies---logical reasoning, mathematical modeling, spatial visualization, and computational fluency---thereby bridging theoretical frameworks with classroom practice. Second, empirical analysis reveals that guidance efficacy is not inherent to method alone but emerges from relational coherence among instructional method, task structure, and classroom discourse; misalignment across these elements consistently attenuates cognitive engagement. Third, the study reconceptualizes teacher guidance as epistemic design: an intentional structuring of disciplinary knowledge access rather than generic cognitive scaffolding. This reframing shifts emphasis from procedural support to the deliberate orchestration of epistemic opportunities---such as problem decomposition pathways, representational flexibility, and justification norms---that instantiate competency development. Collectively, these contributions reposition guidance as a domain-specific pedagogical grammar, demanding precision in method-task-discourse alignment and offering a generative foundation for competency-oriented curriculum implementation, professional development, and future design-based research. The findings underscore that effective guidance operates not at the level of isolated strategies but within integrated, context-sensitive systems of mathematical practice.

6.2. Practical Signposts for Classroom Implementation

Three practical signposts guide effective implementation of thinking guidance aligned with core competencies. First, conduct a rigorous audit of guidance--task alignment by interrogating whether each prompt explicitly invites the reasoning form demanded by the task's epistemic structure---such as deductive inference in proof tasks or covariational reasoning in function modeling. Second, prioritize consistency over frequency: fewer, precisely timed guidance moves embedded within high-leverage instructional moments yield stronger cognitive uptake than numerous, fragmented interventions dispersed across lessons. Third, treat discourse norms as foundational infrastructure rather than auxiliary scaffolds; co-construct explicit, shared expectations for error engagement, justification practices, and iterative idea revision before introducing advanced guidance strategies. These norms must be operationalized through repeated modeling, normative feedback, and student-led reflection cycles---not merely stated. Collectively, these signposts shift emphasis from procedural fidelity to epistemic intentionality, ensuring that guidance functions not as directive correction but as calibrated support for disciplinary reasoning development. Implementation success hinges on sustained attention to structural coherence between task design, teacher move sequencing, and classroom discourse architecture.

6.3. Final Reflection on Competency Realization

Core competencies in middle school mathematics are not acquired through passive exposure or mechanical repetition but realized through sustained, well-guided participation in authentic mathematical practices. The quality of thinking guidance---its intentionality, responsiveness to student reasoning, and coherence with the broader pedagogical ecology---determines whether competency frameworks remain abstract ideals or become embedded features of daily classroom activity. Effective guidance scaffolds students' transitions from procedural execution to conceptual justification, from isolated problem-solving to interconnected mathematical sense-making. It demands continuous teacher judgment, not script-following; adaptive sequencing, not rigid pacing; and epistemic alignment, not fragmented task design. Without such deliberate, context-sensitive mediation, competencies risk reduction to checklist items rather than lived cognitive dispositions. Thus, realization hinges less on curriculum coverage than on the fidelity and sophistication of instructional enactment.

References

1. J. Kilpatrick, "Competency frameworks in mathematics education," in *Encyclopedia of Mathematics Education*. Cham: Springer International Publishing, 2020, pp. 110–113.
2. M. Santos-Trigo, "Problem solving in mathematics education: tracing its foundations and current research-practice trends," *ZDM Mathematics Education*, vol. 56, no. 2, pp. 211–222, 2024.
3. H. M. Doerr, J. B. Ärlebäck, and M. Misfeldt, "Representations of modelling in mathematics education," in *Mathematical Modelling and Applications: Crossing and Researching Boundaries in Mathematics Education*. Cham: Springer International Publishing, 2017, pp. 71–81.
4. S. Schell-Straub, "Mathematics education meets development education: The competency 'mathematical modelling' combined with global skills and competencies in a secondary school project in Germany," *International Journal of Development Education and Global Learning*, vol. 5, no. 1, 2012.
5. B. Srisakda, S. Sujiva, and S. Pasiphol, "Development of indicators of learner's key competencies based on the basic education core curriculum," *Procedia - Social and Behavioral Sciences*, vol. 217, pp. 239–248, 2016.
6. M. Blomhøj and T. H. Jensen, "What's all the fuss about competencies? Experiences with using a competence perspective on mathematics education to develop the teaching of mathematical modelling," in *Modelling and Applications in Mathematics Education: The 14th ICMI Study*. Boston, MA: Springer US, 2007, pp. 45–56.