Article

Backward Instructional Design in High School Mathematics Based on the Alignment of Teaching, Learning, and Assessment

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Abstract: This paper, grounded in the theory of alignment among teaching, learning, and assessment, explores the practical application of backward instructional design in high school mathematics instruction, using the topic "Basic Properties of Probability" from the People's Education Press (A Edition) textbook as a case study. The study first establishes learning objectives through an analysis of curriculum standards and textbook content, identifying overarching concepts, clarifying desired understandings, and formulating essential questions. Assessment evidence is then determined from three dimensions: performance tasks, other forms of evidence, and self-assessment with feedback. Based on the established learning objectives and assessment evidence, instructional activities are developed using the WHERETO framework. Finally, the paper offers instructional reflections from three perspectives: defining measurable learning goals, encouraging peer evaluation through group discussions, and emphasizing timely feedback. This approach aims to enhance instructional coherence, promote deeper student understanding, and strengthen the effectiveness of classroom teaching.

Keywords: alignment among teaching, learning, and assessment; backward instructional design; high school mathematics; basic properties of probability

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1. Introduction

According to the General Senior High School Mathematics Curriculum Standards (2017 Edition) (hereafter referred to as the "Standards 2017"), teaching evaluation should not only focus on students' learning outcomes but also emphasize the learning process. Teachers are encouraged to make timely adjustments to their instructional strategies based on observations of students' learning behaviors and thinking processes [1]. Traditional assessment practices, typically conducted after instruction through homework or examinations, primarily measure students' knowledge acquisition. However, such post-instructional assessments often suffer from delayed feedback, which may lead to misconceptions going unnoticed and reduce opportunities for timely instructional adjustment. To address this issue, the alignment of teaching, learning, and assessment becomes particularly crucial. This alignment emphasizes the interconnected and complementary relationship among the three components, enabling the integration of real-time assessment into classroom instruction. Techniques such as in-class questioning, quizzes, and interactive discussions provide teachers with immediate insights into students' understanding, allowing them to detect issues early and adapt their teaching strategies accordingly. Timely feedback not only enhances instructional responsiveness but also ensures that students can achieve a deeper understanding of the material. Meanwhile, the adoption of backward instructional design effectively integrates intended learning outcomes, assessment tasks, and instructional activities, thereby facilitating a coherent and dynamic connection among teaching, learning, and assessment [2].

2. Backward Instructional Design Process for High School Mathematics Based on the Alignment of Teaching, Learning, and Assessment

Jay McTighe and Grant Wiggins proposed the backward instructional design model, which consists of three stages: identifying desired learning outcomes, determining acceptable evidence, and planning learning experiences and instruction [3]. Building upon this framework, the backward design approach for high school mathematics — grounded in the alignment of teaching, learning, and assessment — has been further refined to ensure a high degree of consistency among teaching practices, student learning processes, and assessment strategies, all of which are closely aligned with the intended learning goals.

2.1. Identifying Desired Learning Objectives

First, it is essential to analyze the curriculum standards and textbook content in order to identify the core competencies and big ideas embedded in the instructional material. This ensures that the learning objectives are aligned with the national curriculum requirements. Second, teachers need to clarify the expected understandings of students by specifying the key knowledge, essential skills, and fundamental concepts that students are expected to acquire. Third, it is important to formulate essential questions — open-ended, thought-provoking questions that guide students toward inquiry-based learning, stimulate deep thinking, and encourage them to apply their knowledge to solve real-world problems. Finally, specific, measurable, and assessable learning goals should be defined to provide clear direction for student learning and minimize the influence of teacher subjectivity in instruction. The specific steps of Stage One and their corresponding alignment with teaching, learning, and assessment are illustrated in the following Table1:

Table 1. The Specific Process of Stage One and Its Alignment with Teaching, Learning, and Assessment.

	Process	Manifestation of the Alignment among Teaching, Learning, and Assessment
Identifying Desired Learning Objectives	Analyzing curriculum standards and textbook content to extract big ideas	Teaching: Instructional content is strictly designed in accordance with the curriculum standards and is centered around the big ideas. Learning: Student learning activities are developed based on core competencies and key knowledge points. Assessment: Evaluation criteria are aligned with core competencies to ensure that assessment remains focused on the intended learning objectives.
	Clarifying expected understandings	Teaching: Teachers ensure that instructional content is directly aligned with the desired understandings. Learning: Learning activities are designed to guide students toward achieving the expected understandings. Assessment: Assessment tasks (e.g., performance tasks, open-ended questions) evaluate the depth of students' conceptual understanding.
	Formulating essential questions	Teaching: Teachers organize classroom instruction around essential questions to guide students toward deep thinking. Learning: Students actively construct knowledge through group discussions and performance tasks.

	Assessment: Assessment tasks are designed to determine whether students can answer these essential questions and demonstrate the depth of their understanding.
Defining specific learning objectives	Teaching: Instructional goals set by the teacher must be specific and clearly defined, rather than vague or overly general. Learning: Student learning tasks should directly support the achievement of these goals. Assessment: All forms of assessment should be strictly aligned with the learning objectives, avoiding the evaluation of unrelated content.

2.2. Determining Assessment Evidence

Real-world contexts serve as an effective pathway for cultivating students' core mathematical competencies [3]. First, tasks should be designed within authentic situations so that students can apply their mathematical knowledge to solve real problems. Second, in addition to performance-based assessments, a variety of assessment methods should be employed to gain a comprehensive understanding of student learning — such as classroom questioning, group discussions, and teacher observations. Finally, students should engage in self-assessment and peer feedback, enabling them to reflect on their own learning processes and make necessary adjustments. The specific steps of Stage Two and their corresponding alignment with teaching, learning, and assessment are shown in the following Table2:

Table 2. The Specific Process of Stage Two and Its Alignment with Teaching, Learning, and Assessment.

	Process	**Manifestation of the Alignment among Teaching, Learning, and Assessment**
Determining Assessment Evidence	designing performance tasks	Teaching: Classroom instruction is designed around the knowledge and skills required for completing the performance tasks. Learning: Students actively engage in learning and construct knowledge through tasks set in real-world contexts. Assessment: Evaluation focuses not only on students' mastery of knowledge but also on their problem-solving abilities.
	collecting additional evidence	Teaching: Teachers continuously collect evidence throughout the instructional process to adjust teaching strategies in a timely manner. Learning: Student learning tasks are aligned with assessment methods to ensure consistency with learning goals. Assessment: Assessment is not limited to summative tests; it is a dynamic process that incorporates multiple dimensions and levels.
	self-assessment and feedback	Teaching: Teachers guide students to reflect and adjust their learning strategies during the instructional process. Learning: Students actively engage in the assessment process, developing their metacognitive abilities. Assessment: Assessment is no longer a one-sided judgment by the teacher, but an interactive process that enhances learning outcomes.

2.3. Designing Learning Activities

Teachers should refer to the WHERETO framework (see Table 3 for its components and implementation strategies) when designing appropriate learning activities based on the predetermined learning objectives and assessment tasks. To promote alignment among teaching, learning, and assessment, instruction should follow a structured sequence of "Objective 1 → Instruction 1 → Assessment 1 → Objective 2 → Instruction 2 → Assessment 2 …", ensuring that each instructional segment is directly linked to specific goals and corresponding evaluations.

Table 3. The WHERETO Framework and Its Corresponding Implementation Strategies.

Components of the WHERETO Framework	Specific Implementation Strategies
W (Where)	Clarify learning objectives so that students understand what they are expected to learn.
H (Hook)	Stimulate students' interest through contextual scenarios and problem-based introductions.
E-1 (Explore)	Provide opportunities for exploratory learning that enable students to construct knowledge independently.
R (Rethink)	Encourage students to reflect on and adjust their learning strategies.
E-2 (Equip)	Guide students to engage in self-assessment and make necessary adjustments.
T (Tailor)	Adjust learning pathways according to students' individual needs to support differentiated learning.
O (Organize)	Organize learning activities in alignment with the sequence of cognitive development to facilitate knowledge transfer.

3. Backward Instructional Design of the High School Mathematics Topic "Basic Properties of Probability" Based on the Alignment of Teaching, Learning, and Assessment

3.1. Identifying Desired Learning Objectives

3.1.1. Analyzing Curriculum Standards and Textbook Content to Extract Big Ideas

According to the Standards 2017, students are expected to "understand the properties of probability through examples and master the operational rules for calculating the probabilities of random events". Therefore, teachers are required to use real-life contexts to help students comprehend and internalize the basic properties of probability, thereby fostering their ability to both understand and apply probabilistic concepts.

An analysis of the textbook reveals that the core concept of this lesson is the basic properties of probability, which include non-negativity, the total probability summing to 1, the addition rule for mutually exclusive events, and the relationship between complementary events. At the beginning of the lesson, the textbook offers a guiding prompt: students are encouraged to draw analogies from the properties of exponential functions to develop an analytical perspective on the properties of probability. Teachers can leverage this analogy by highlighting the parallel structure between the study of probability and that of functions, guiding students to explore the properties of probability by referring to their understanding of exponential functions. Through specific contextualized examples, students are led to generalize and abstract the underlying principles behind each probabilistic property.

In addition, this lesson aims to help students develop computational skills in probability by mastering the related operational rules. Through engagement with these basic properties, students can begin to form a problem-solving approach that involves analyzing relationships between events and applying probabilistic properties to solve real-world problems.

In summary, the big idea of the lesson "Basic Properties of Probability" can be defined as: "The fundamental laws of probability and their practical applications."

3.1.2. Clarifying Expected Understandings

After identifying the big idea, teachers should clearly define what students must know, what skills they should acquire, and what core concepts they are expected to understand by the end of the lesson (as shown in Table 4).

Table 4. Students' Expected Understandings.

Clarifying expected understandings	**Key knowledge points**	**The six basic properties of probability, including non-negativity and the addition rule for mutually exclusive events.**
	Skills to be acquired	Probability calculation skills: Students will be able to proficiently apply the basic properties of probability to solve probability problems in various contexts. Simplifying complex probability problems: Students will be able to identify and use the addition rule, the properties of complementary events, and set inclusion relationships to simplify complex probability calculations. Reasoning and verification: Students will be able to verify conjectures through mathematical reasoning, understand the logical derivation of probability properties, and apply these reasoning methods to probability calculations in real-world scenarios.
	Core concepts to be understood	Basic properties of probability: Students should understand how probability is defined and how it applies to events within a sample space. Relationships between events: Students should understand and be able to apply the relationships among different types of events (e.g., mutually exclusive, complementary, and inclusive relationships).

3.1.3. Formulating Essential Questions

Essential questions are key prompts that help students grasp core concepts and guide their thinking processes. Based on the analysis of the textbook, the following four essential questions were identified:

1) What are the basic properties of probability?
2) How do relationships between events affect their probabilities?
3) How can the basic properties of probability be applied to real-world problems?
4) How can the basic properties of probability be proven?

3.1.4. Defining Specific Learning Objectives

Based on the analysis of the big idea, expected understandings, and essential questions, the following specific learning objectives can be formulated:

Students will be able to analyze the properties of probability by drawing analogies with the study of function properties.

Students will be able to understand, derive, and verify the six basic properties of probability, and explain the underlying meanings and applicable contexts of these properties.

Students will be able to critically analyze problems, identify when and how to apply specific probability properties, and understand how these properties can simplify the process of probability calculation.

3.2. Determining Assessment Evidence

After defining the learning objectives, teachers should design a variety of assessment evidence (as shown in Table 5) to determine whether students have achieved these objectives and to inform the planning of subsequent instructional activities. Predefining assessment evidence helps ensure alignment between learning goals and evaluation criteria. It also enables teachers to focus time and resources on the most critical aspects of student learning outcomes, thereby improving instructional efficiency.

Table 5. Assessment Evidence.

Stage 2 — Determining Assessment Evidence
Performance Task: You are the coordinator of a classroom activity. Today, your class is conducting a simple mathematical experiment involving coin tossing to explore how the probability of all coins landing heads-up changes as the number of coins increases. Together with your classmates, you will toss 1, 2, 3, ..., n coins, recording the frequency of all heads for each case. You are expected to derive the theoretical probability using mathematical formulas and describe the trend using a function expression. Please write an experimental report addressing the following questions: How does the probability of getting all heads change as n increases? Use data and graphs to describe the trend. What type of mathematical function does this probability trend resemble as n increases? Write the corresponding mathematical expression. Can you provide other real-world probability phenomena that follow a similar pattern? Other Evidence: Classroom Observation: Observe whether students can clearly explain the six basic properties of probability; whether they can verify these properties through derivation; and whether they can identify which property to apply in real-world problem-solving contexts. Oral Questioning: Assess whether students can clearly articulate the basic properties of probability, understand the scenarios in which they are applied, and accurately derive or justify these properties. For example, evaluate whether students can provide reasonable real-life examples (e.g., coin tosses, dice rolls) to explain the non-negativity of probability. Paper-and-Pencil Tests: Determine whether students are able to solve problems involving the application of each of the six basic probability properties. Self-Assessment and Feedback: Summarize the knowledge framework using a concept map or outline. Reflect on the lesson, identify any unresolved questions, and engage in discussions with group members to share and clarify understandings.

To enhance the effectiveness of performance tasks as an assessment tool, specific task requirements and a corresponding scoring rubric were designed (as shown in Table 6).

Table 6. The Performance Task Requirements and Corresponding Scoring Rubric.

Task requirements
You are required to complete the following tasks: Data Collection and Experimental Design: Conduct a coin-tossing experiment in class. Each student takes turns tossing 1, 2, 3, ..., n coins, and records the frequency of all coins landing heads-up in each case. Theoretical Calculation: Calculate the theoretical probability of all n coins landing heads-up, and compare the result with the experimental data.

Data Visualization: Record the frequency of heads for each trial and create a line graph or scatter plot to visually illustrate how the probability changes with increasing n.
Mathematical Modeling: Analyze the trend in the data and identify a mathematical expression for the probability function. Explain how this function relates to an exponential function.
Application and Extension: Can you identify other real-world problems that follow a similar probability pattern?

Scoring rubric	
Assessment criteria	**Rating scale**
Construction of the Probability Function Model (10 points)	The student accurately collects data, creates tables, and correctly plots the curve showing the change in probability.
Analysis of the Probability Function (10 points)	The student identifies and explains the exponential decay trend of the probability and correctly writes the corresponding mathematical formula.
Mathematical Explanation Ability (10 points)	The student is able to explain the nature of the probability change using the mathematical principles of exponential functions.
Application and Extension to Real-World Contexts (10 points)	The student provides other real-life examples of probability phenomena that exhibit exponential decay and explains the underlying mathematical pattern.

3.3. Designing Learning Activities

After determining the assessment evidence, it is essential to design targeted learning activities that support the completion of the assessment tasks. In this process, teachers are expected to develop activities that address students' actual learning needs and promote the development of their skills and competencies. These activities should be closely aligned with the assessment goals to ensure instructional coherence, while also serving as a basis for instructional reflection and improvement. The alignment between instructional objectives, learning activities, and assessment is illustrated in Table 7.

Table 7. Learning Activities and Assessment.

Objective 1	**Students will be able to analyze the properties of probability by drawing analogies with the study of function properties.**	
Instruction 1	Activity1-1	Conduct a Mathematical Experiment: Students work in groups and take turns tossing 1, 2, 3, and 4 coins, recording the frequency of all coins landing heads-up each time. Observation Prompt: As n increases, how does the probability of all heads change? Does this trend resemble any known mathematical function? — H
	Activity 1-2	Based on the probability of all n coins landing heads-up, guide students to recall the properties of exponential functions. Encourage them to think about which perspectives can be used to explore the properties of probability. — W, E-1
Assessment 1	Ask students to write an experimental report and evaluate it using the scoring rubric for the performance task. — O, E-2	
Objective 2	**Students will be able to understand, derive, and verify the six basic properties of probability, and explain the underlying meanings and applicable contexts of these properties.**	
Instruction 2	Activity 2-1	Present an incorrect scenario, such as "The probability of a coin landing heads-up is -0.2," and guide students to think critically about why

		this statement is invalid. Use this discussion to introduce the non-negativity property of probability. — H
	Activity 2-2	Through the mathematical experiment, guide students to discuss and conclude that the special point (0, 1) on an exponential function can be analogized to the fact that the probability of a certain event is 1. Similarly, the probability of an impossible event being 0 corresponds to the behavior of the exponential function, where the probability approaches 0 as n approaches infinity. — E-1
	Activity 2-3	Use the example of rolling a die to guide students in calculating the probabilities of the following events: "the number rolled is greater than 3", "the number rolled is less than 3", and "the number rolled is greater than 3 or less than 3". Use these calculations to verify the addition rule of probability. — E-1
	Activity 2-4	Have students discuss in groups how to apply the addition rule of probability in the following two cases: Drawing one card from a standard deck and finding the probability of getting a heart or a spade. Randomly selecting one student from the class and determining the probability of selecting a boy or a girl. — O
	Activity 2-5	Continue using the die-rolling example to prompt students to think about the relationship between the probabilities of the complementary events "rolling a number greater than or equal to 3" and "rolling a number less than 3". Use this discussion to introduce Property 4 — the probability relationship between complementary events. — E-1, R
	Activity 2-6	After learning the properties of mutually exclusive and complementary events, students are encouraged to explore the relationship between another special type of event: inclusion. In groups, students will create a concept map that illustrates the inclusion relationship and the corresponding definition of probability comparison. They are also guided to understand these concepts by drawing an analogy with the monotonicity of functions. — T, O
	Activity 2-7	Continue using the die-rolling example to calculate the probability of the union of two events: Event A —"The number rolled is greater than 3" Event B — "The number rolled is even" Through this example, guide students to recognize that when Events A and B are not mutually exclusive, the probability of their union must account for the intersection of the two events. This leads to the introduction of Property 6, the general addition rule. During the learning process, help students realize that Property 3 (the addition rule for mutually exclusive events) is actually a special case of Property 6. — E-1
	Assessment 2	Classroom observation：Observe whether students are able to clearly explain the underlying meaning and application scenarios of each probability property during classroom discussions. Additionally, assess whether they can understand and apply these properties through real-life examples. — E-2 Oral questioning：Why can we directly apply the addition rule without subtracting the intersection when Events A and B are mutually exclusive? — R Paper-and-Pencil Test：Design computational problems focused on the application of probability properties, particularly those involving the addition rule, complementary events, and inclusion relationships. — E-2

Objective 3	Students will be able to critically analyze problems, identify when and how to apply specific probability properties, and understand how these properties can simplify the process of probability calculation.	
Instruction 3	Activity 3-1	The teacher presents complex probability problems — for example, calculating the union probability of two mutually exclusive events or the union of multiple events — to encourage students to analyze which probability properties can help simplify the calculations. Students then exchange their solutions with peers, engage in peer evaluation, and reflect on whether alternative solution strategies exist or whether the calculation can be further simplified. — R, O
	Activity 3-2	Guide students to reflect on how the properties of probability can be applied in different disciplines or real-world contexts. — O
Assessment 3	Classroom observation：Determine whether students can clearly explain why they chose a particular probability property and how they applied it to simplify the problem. — E-2	
	Oral questioning: You chose the multiplication rule to calculate the probability of two events occurring together. Please explain how the multiplication rule helps simplify the calculation, and provide an example of a situation in which this rule should be used. — E-2, T	
Lesson summary	Ask students to summarize the knowledge learned in this lesson using a concept map. After completing their maps, students should exchange them within their groups to identify any missing information and help each other improve and complete their understanding. — T	
	Have students identify and summarize the learning difficulties they encountered during the lesson, and discuss possible solutions with their group members. — E-2, O	

4. Conclusion

4.1. Defining Measurable Learning Objectives

In teaching design based on the alignment of teaching, learning, and assessment, it is essential to establish measurable learning objectives. First, clear and measurable goals allow teachers to develop explicit assessment criteria and conduct objective evaluations that more accurately reflect students' learning levels. Second, such objectives support teachers in providing timely feedback and making instructional adjustments during the teaching process. Finally, measurable learning goals not only assist teachers in evaluating student performance, but also encourage student self-assessment and reflection.

4.2. Encouraging Peer Assessment through Group Discussions

Research shows that peer assessment has a moderately positive impact on primary and secondary school students' learning, with the greatest effect observed among high school students [4]. The alignment of teaching, learning, and assessment emphasizes that teachers should attend not only to learning outcomes but also to interaction and feedback during the learning process. In addition to arranging direct peer assessment in class, teachers can create opportunities for peer feedback through strategies such as group discussions and collaborative tasks, offering feedback on performance tasks, and exchanging concept maps.

4.3. Emphasizing Immediate Feedback

Traditional forms of assessment, such as final exams or homework, often provide delayed feedback, which may allow students to develop misconceptions. In contrast, immediate feedback — delivered through in-class questioning, quizzes, and interactive discussions — helps students continually revise their understanding during the learning process. By receiving timely feedback in class, students can recognize the areas in which they struggle and make necessary adjustments to their learning strategies with the support of teachers or peers.

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