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Article

Backward Teaching Design for Middle School Mathematics Based on Teaching-Learning-Assessment Coherence

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Abstract: Based on the theory of teaching-learning-assessment coherence and combined with the concept of backward teaching design, this paper conducts systematic teaching design and analysis around the middle school mathematics lesson "Opposite Numbers". First, through analysis of teaching-learning-assessment coherence and the backward teaching design process, we optimize and improve upon the backward teaching design framework to obtain a high school mathematics backward teaching design process based on teaching-learning-assessment coherence. Then, starting from three stages: "determining expected goals", "determining appropriate assessment evidence", and "designing learning activities", we construct an integrated teaching model of "objectives-teaching-evaluation". Through the teaching design process of "Opposite Numbers", we explore how to achieve organic integration of teaching and evaluation to improve teaching quality. Finally, we reflect on the teaching from three aspects: "designing rich, multi-layered activities", "focusing on differentiated instruction", and "designing concept formation processes that align with cognitive patterns".

Keywords: teaching-learning-assessment coherence; backward teaching design; opposite numbers

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1. Introduction

The "General High School Mathematics Curriculum Standards (2017 Edition, 2020 Revision)" requires that high school mathematics teaching should focus on and evaluate both students' knowledge learning processes and the evaluation of core competencies integrated within knowledge [1]. Teaching-learning-assessment coherence, starting from developing students' core competencies, establishes scientific and comprehensive learning objectives and emphasizes diverse evaluation methods, making "teaching," "learning," and "assessment" highly consistent around learning objectives. However, in mathematics teaching, teachers' instruction and evaluation are sometimes influenced by subjective factors, and "teaching," "learning," and "assessment" may experience problems of "mutual separation" and "insufficient integration." To address these issues, backward teaching design is an effective approach that can effectively integrate expected objectives, evaluation tasks, and teaching activities to achieve mutual integration of teaching, learning, and assessment [2]. Therefore, it is necessary to conduct backward teaching design based on teaching-learning-assessment coherence.

2. Analysis of Instructional Design Process

2.1. Teaching-Learning-Assessment Coherence

Teaching-learning-assessment coherence refers to "teacher instruction," "student learning," and "instructional evaluation" all revolving around shared, core competency-oriented learning objectives [3]. Teaching segments are developed for each objective, and evaluation tasks are separately set for each teaching segment. Teachers can use evaluation to determine whether students have achieved learning objectives and make decisions for the next stage of teaching [3].

2.2. Backward Teaching Design

Backward teaching design is a learning objective-oriented instructional design method that emphasizes centering on clear learning outcomes and planning instructional content and activities through reverse thinking. Backward teaching design consists of three stages: "determining expected goals," "determining appropriate assessment evidence," and "designing learning experiences" [4]. First, clarifying teaching objectives is the core of backward teaching design, and teachers need to clarify big ideas and core tasks. After clarifying objectives, teachers need to assess whether students' performance meets predetermined standards through designing tests, assignments, performance tasks, and student self-assessment. Finally, teachers need to design specific teaching activities and learning processes based on learning objectives and assessment requirements.

2.3. High School Mathematics Backward Teaching Design Process Based on Teach-Ing-Learning-Assessment Coherence

Teaching-learning-assessment coherence emphasizes the "objective → teaching → evaluation" teaching model, which aligns with the logic of "starting from objectives, designing assessment, then planning teaching" in backward teaching design. Therefore, we improve the backward teaching design process to make it more prominently reflect teaching-learning-assessment coherence.

2.3.1. Determining Expected Goals

Big ideas are important pathways for developing students' mathematical core competencies [5]. Therefore, we first analyze curriculum standards and textbooks to identify the core competencies students need to develop and big ideas that can promote knowledge transfer. Then, based on core competencies and big ideas, we determine expected teaching objectives and preset "what students need to understand" and "what students can do," thereby reducing the impact of teachers' subjective factors on teaching and improving "teaching-learning" consistency.

2.3.2. Determining Appropriate Assessment Evidence

"Authentic contexts" are effective pathways for cultivating students' mathematical core competencies [6]. Establishing performance tasks based on authentic contexts helps students understand knowledge and improve problem-solving abilities. Additionally, teachers can use observation, dialogue, testing, and student self-assessment to evaluate students' understanding of knowledge, thereby improving "teaching-assessment" consistency.

2.3.3. Designing Teaching Activities

Finally, teachers need to refer to WHERETO elements (as shown in Table 1) [4] and develop corresponding learning activities based on preset learning objectives and assessment tasks. Teachers set evaluation activities after each learning activity to improve "learning-assessment" consistency.

Table 1. Content of WHERETO Elements.

Element	Content
W	Clearly explain to students the core content of learning and the practical significance of learning this knowledge.
H	Attract students' attention through interesting questions, activities, or cases.

E	Enable students to actively participate in learning through hands-on practice and exploration activities.
R	Have students reflect on what they have learned and reconsider their understanding and perspectives.
E	Guide students in self-assessment and adjustment.
T	Provide diverse learning pathways and adjust instructional design according to students' different abilities.
O	Optimize teaching structure to ensure learning activities progress step by step and are interconnected.

3. Backward Teaching Design of "Opposite Numbers" Based on Teaching-Learning-Assessment Coherence

3.1. Determining Expected Goals

3.1.1. Analyzing Textbooks and Extracting Big Idea

The textbook adopts a cognitive pathway from concrete to abstract, naturally introducing the algebraic concept of opposite numbers through the geometric idea of two points on a number line that are equally distant from the origin. This design embodies the mathematical thinking of combining numbers and shapes, allowing students to understand abstract mathematical concepts through intuitive geometric relationships. Symmetry in mathematics is a fundamental geometric and algebraic relationship. Opposite numbers reflect symmetry about the origin on the number line, and this symmetry is a widely occurring concept in mathematics. The concept of opposite numbers reflects the mathematical idea of reasoning in reverse. Through the "-" operation, we can obtain the opposite of a number, laying the foundation for subsequent learning of subtraction operations and solving equations. From specific number pairs (such as 2 and -2), we derive the abstract concept of opposite numbers (a and -a), illustrating the mathematical reasoning process from the particular to the general.

Based on the above analysis of the textbook, we can derive the big ideas: "symmetry," "combination of numbers and shapes","reverse thinking in operations", and "mathematical abstraction and generalization".

3.1.2. Clarifying Expected Understanding

After obtaining the big ideas, teachers should clearly set the knowledge points students must know, skills they must master, and core concepts they must understand after completing this lesson (as shown in Table 2).

Table 2. Students' Expected Understanding.

Clarifying Expected Understanding	Knowledge Points	Definition of opposite numbers, representation methods of opposite numbers, geometric meaning of opposite numbers, special property of 0, criteria for judging opposite numbers, sign operation rules for opposite numbers, symmetry properties of opposite numbers, simplification of multiple signs.
	Skills to Master	• Accurately judge whether two numbers are opposite to each other. • Correctly write the opposite of any given number. • Accurately mark the positions of numbers and their opposites on the number line. • Simplify mathematical expressions containing multiple signs. • Distinguish between negative signs and opposite number signs.

	• Identify symmetric relationships of points on the number line. • Infer properties of original numbers given opposite number relationships. • Correctly apply the concept of opposite numbers in problem contexts.
Core Concepts to Understand	• The concept of symmetry. • The idea of combining numbers and shapes• • Mathematical abstract thinking.

3.1.3. Setting Essential Questions

"Essential questions" are key questions that help students understand core concepts and guide student thinking. Through textbook analysis, we obtain four essential questions: "On the number line, what kind of two points have special positional relationships?" "Why did mathematicians create the concept of 'opposite numbers'? What problems does it solve?" "In real life, what situations can be described using opposite numbers?" "What information about numbers can distance and position on the number line tell us?"

3.1.4. Formulating Specific Learning Objectives

Based on analysis of "big ideas," "expected understanding," and "essential questions," we can obtain specific learning objectives:

1) Understand the definition of opposite numbers, accurately judge whether two numbers are opposite to each other, and accurately mark the positions of numbers and their opposites on the number line.
2) Through observing points equidistant from the origin on the number line, experience the concept formation process from concrete to abstract, use the number line to intuitively understand the concept of opposite numbers, and appreciate the internal connection between geometric intuition and algebraic operations.
3) Through number line exploration activities, stimulate interest in learning mathematics and enthusiasm for exploring mathematical patterns, and help students recognize the structural symmetry in opposite numbers and the efficiency of mathematical notation.

3.2. Determining Assessment Evidence

After determining learning objectives, we must immediately consider assessment evidence for whether students have truly achieved these objectives (as shown in Table 3). This avoids the problem of misalignment between instruction and learning outcomes and ensures high alignment between evaluation and objectives. The assessment design process in Stage 2 is actually a further clarification and deepening of the objectives in Stage 1. In thinking about "how to assess," teachers often discover the need to adjust or improve original learning objectives.

Table 3. Assessment Evidence.

Stage 2 — Determining Assessment Evidence
You are a young mathematical explorer who has come to the mysterious "Kingdom of Symmetry." This kingdom has a peculiar rule: there is a magical mirror (origin) at the center of the kingdom, and any number through this mirror will produce its "mirror partner." The king tells you that only by cracking the pattern of this "mirror code" can you obtain the key to enter the mathematical treasure. (1) On the given "kingdom map" (number line), the king has placed some number castles: +3, -5, +2, -1, 0, +4.5, -2.5. First, you need to find the "mirror partner" for each castle

and mark their positions on the map. Then, describe in your own words what kind of two numbers can become "mirror partners." (2) The king has given you a special observation tool (which can be a physical number line or digital tool). Choose any positive number a, find all castle positions on the number line that are at distance a from the "magical mirror" (origin), observe what special relationships these positions have, and create a "mirror transformation rule" that can help other explorers quickly find the mirror partner of any number. (3) As a new resident of the Kingdom of Symmetry, the king invites you to design a mathematical artwork that embodies "mirror aesthetics." You need to design a symmetric pattern decorated with mirror partner number pairs (including at least 8 pairs of mirror partners).
Performance Task Scoring Rubric: • Part 1: Finding Mirror Partners (30 points) (1) Accuracy (20 points): Correctly find the opposite numbers of 7 numbers. (2) Position marking (10 points): Accurately mark positions on the number line. • Part 2: Pattern Exploration (35 points) (1) Distance observation (15 points): Correctly find two positions at distance a from the origin. (2) Pattern description (20 points): Able to explain characteristics of mirror partners in own words. • Part 3: Symmetric Pattern Design (35 points) (1) Quantity requirement (15 points): Include 8 or more mirror partner number pairs. (2) Accuracy (10 points): Mirror partner pairing is correct. (3) Aesthetic creativity (10 points): Pattern design embodies symmetric aesthetics.
Other Evidence: 1. Classroom observation: Students' enthusiasm for participating in discussions and accuracy of observations; effectiveness and participation in group cooperation. 2. Q&A: Logic of students' reasoning processes and accuracy of conclusions; whether students can identify and correct common errors; clarity of student presentations and quality of peer evaluations. 3. Paper-and-pencil tests: Whether students can accurately annotate numbers on the number line and discover patterns.
Self-Assessment and Feedback: 1. Summarize knowledge framework diagrams. 2. Reflect on this lesson, summarize unresolved problems, and communicate with group members.

3.3. Designing Teaching Activities

The third stage is the concrete implementation of the first two stages, transforming determined learning objectives and assessment methods into operational teaching activities (as shown in Table 4). Based on learning objectives determined in the first stage and assessment standards in the second stage, designing specific learning experiences that can help students achieve objectives ensures high consistency among teaching activities, assessment methods, and learning objectives, avoiding teaching that deviates from predetermined goals.

Table 4. Learning Activities.

Objective 1	Understand the definition of opposite numbers, accurately judge whether two numbers are opposite to each other, and accurately mark the positions of numbers and their opposites on the number line.	
Instruction 1	Activity1-1	Through "learning objective cards," clearly communicate specific objectives to be achieved in this lesson to students, have students restate in their own words, and predict what problems they can solve after learning. W
	Activity1-2	Display a thermometer, ask "What is the relationship between 5°C above zero and 5°C below zero?" Students observe thermometer scales, discuss relationships between positive and negative temperatures, triggering thinking about the concept of "opposite." H
	Activity1-3	Mark numbers on the number line. Given a number line, have students mark +3 and -3, measure their distances from the origin, observe their positional relationships, and try to find more such number pairs. E
Assessment 1	Classroom observation: Whether students can accurately understand learning expectations. Q&A: Students' enthusiasm for participating in discussions and accuracy of observations. Paper-and-pencil tests: Whether students can accurately annotate and discover patterns.	
Objective 2	By observing points equidistant from the origin on the number line, students can experience the concept formation process from concrete to abstract, use the number line to intuitively understand the concept of opposite numbers, and appreciate the internal connection between geometric intuition and algebraic operations.	
Instruction 2	Activity 2-1	Observe specific number pairs: 2 and -2, 5 and -5, 1.5 and -1.5. Ask: What common characteristics do these number pairs have? Guess: For any positive number a, which other number on the number line has the same relationship with it? Then verify the guess and form the definition of opposite numbers. E-2
	Activity 2-2	Help students improve their understanding of the concept of opposite numbers through a question sequence. Think: "Is -a necessarily a negative number?" Verify with examples: When a = -3, -a = 3. Reflect: Why might there be such misunderstanding? Finally, improve understanding of the concept of opposite numbers. R, T
	Activity 2-3	Have students use a ruler to measure the distance from the origin to themselves. Think: Which other point is also 0 units away from the origin? Analyze: 0 is neither positive nor negative; what does its "different sign" mean? Finally conclude: The opposite of 0 can only be 0 itself. E-1
Assessment 2	Classroom observation: Whether students can discover the special property of 0 through measurement and understand the mathematical reasoning that the opposite of 0 is 0. Q&A: Logic of students' reasoning processes and accuracy of summarized concepts.	
Objective 3	Through number line exploration activities, stimulate interest in learning mathematics and enthusiasm for exploring mathematical patterns, and	

		appreciate the symmetric beauty embodied in the concept of opposite numbers and the concise beauty of mathematical symbols.
Instruction 3	Activity 3	Students choose favorite colors and patterns, create symmetric patterns on the number line, label with opposite numbers, write creative descriptions for their works explaining their feelings about symmetric beauty, and share artistic works and learning insights within groups. H, O
Assessment 3		Classroom observation: Creativity of student works and expression of feelings about symmetric beauty; clarity of displayed works and quality of peer evaluations.
Lesson summary		Summarize the knowledge learned in this lesson in the form of a framework diagram, then have group members exchange diagrams and help each other fill gaps. T Summarize learning difficulties encountered in this lesson and discuss solutions with group members. E-2, O丨

4. Conclusion

4.1. Designing Rich, Multi-Layered Activities

The progression from real-life contexts such as thermometers, to geometric operations on number lines, and finally to abstract symbolic manipulation, reflects the cognitive development from concrete to abstract thinking. We skillfully combine different types of activities such as observation, measurement, annotation, and creation, allowing students to gradually construct the concept of opposite numbers through multi-sensory participation. However, the cognitive demands of these activities need to be increased. The current design mainly stays at the concept understanding and basic application levels, lacking sufficient extensibility and exploratory tasks.

4.2. Focusing on Differentiated Instruction

Ideal differentiated design should provide tiered tasks for students at different levels. For example, the basic group could focus on identification and basic properties of opposite numbers, the improvement group could explore the geometric meaning of opposite numbers on the number line, and the extension group could study generalizations and applications of the opposite number concept. At the same time, we should provide diverse learning tools and means of expression. In evaluation, we need to establish diversified evaluation standards that allow students to demonstrate their learning outcomes in different ways. Some students excel at oral expression, some are better suited to written assignments, and others perform better in practical operations. Only in this way can we truly achieve individualized instruction and enable every student to make meaningful progress based on their existing foundation.

4.3. Designing Concept Formation Processes That Align with Cognitive Patterns

This teaching design follows basic cognitive patterns of mathematical concept formation well. Starting from concrete examples of thermometers, gradually transitioning to geometric representation on number lines, and finally abstracting the algebraic concept of opposite numbers, it reflects the cognitive development path from concrete to abstract and from perceptual to rational. However, there are still some areas that need optimization in key links of concept formation. For example, the transition from thermometer to number line could be more natural and smooth, as it might feel abrupt to some students. In the concept construction stage, although we addressed the important cognitive point that "-a is not necessarily negative," we could design more thoroughly for such areas prone to cognitive conflict, intentionally introducing common misconceptions to help students deepen their understanding through guided error correction.

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