Financial Economics Insights

Article

Research on High-Frequency Stock Pair Trading Strategy Based on MS-GARCH Model

Jun Dong [1,*]

[1] Shenyang Yingqi Technology Co., Ltd, Shenyang, Liaoning, China
* Correspondence: Jun Dong, Shenyang Yingqi Technology Co., Ltd, Shenyang, Liaoning, China

Abstract: With the rapid development of high-frequency trading, pair trading, a classic statistical arbitrage strategy, has gradually become a widely applied trading method in the market. This paper focuses on the construction and application of high-frequency stock pair trading strategies based on the MS-GARCH (Markov Switching Generalized Autoregressive Conditional Heteroskedasticity) model. First, we select suitable stock pairs for pair trading by performing cointegration tests on high-frequency stock data. Then, the MS-GARCH model is used to model the price volatility of these stock pairs, and a high-frequency pair trading strategy is designed based on this model. Through simulation experiments, the effectiveness and robustness of the strategy under different market conditions are verified. The research results show that the MS-GARCH model can effectively capture the volatility characteristics of the market, helping investors achieve better returns and risk control in high-frequency trading. Finally, the paper discusses the potential applications and improvements of high-frequency stock pair trading strategies.

Keywords: MS-GARCH model; high-frequency trading; pair trading; stock volatility; cointegration test; statistical arbitrage

Published: 20 December 2024

1. Introduction

High-frequency trading (HFT) is an advanced trading method in financial markets that uses high-speed computing and algorithmic trading to execute a large number of transactions in a very short period. With the increasing complexity of financial markets, high-frequency trading has become a significant force in the market. Especially in the stock market, traders use high-frequency data (such as minute-level, second-level, or even higher frequency data) for short-term trading, aiming to capture small price fluctuations and generate profits. Pair trading, a classic statistical arbitrage strategy, allows for the construction of trading pairs between different stocks, using the monitoring and prediction of price differences to identify arbitrage opportunities. The basic idea of pair trading is to select two stocks with a strong cointegration relationship and create a market-neutral trading portfolio, thus avoiding overall market risk and relying solely on the price relationship between the stocks to generate profits. However, in traditional pair trading models, it is often assumed that the volatility of assets is constant, ignoring the changes in market volatility. This assumption can lead to underperformance in high-frequency trading environments. To address this issue, this paper introduces the MS-GARCH (Markov Switching Generalized Autoregressive Conditional Heteroskedasticity) model into high-frequency stock pair trading. The MS-GARCH model can capture volatility changes under different market states by incorporating a state-switching mechanism and effectively simulate the dynamic variations in volatility and risk in the financial market. This model not only improves the accuracy of price volatility modeling but also provides more precise risk control and trading signals for high-frequency stock pair trading strategies. The goal of this research is to conduct in-depth analysis and experimental validation of high-frequency stock pair trading strategies using the MS-GARCH model. First, high-frequency stock data is selected, and cointegration tests are performed to filter suitable stock pairs for pair trading. Then, the MS-GARCH model is used to model the volatility of these stock pairs, and a trading strategy is further developed. Backtesting is conducted to analyze the performance of the strategy in actual trading. Ultimately, this paper aims to provide a new theoretical framework and technical path for high-frequency stock pair trading practice and offer market participants a more robust and effective trading strategy [1].

2. Literature Review

2.1. Overview of the MS-GARCH Model

The MS-GARCH (Markov Switching Generalized Autoregressive Conditional Heteroskedasticity) model is an extension of the traditional GARCH model. By incorporating the Markov state-switching mechanism, it significantly enhances the ability of conditional heteroskedasticity models to capture the dynamic volatility changes in financial time series. In financial markets, volatility often exhibits significant variations across different periods. For instance, during economic crises, the market may experience high volatility, while in stable periods, it may experience low volatility. The traditional GARCH model cannot capture this dynamic state switching. However, the MS-GARCH model describes market states as latent variables and uses a Markov chain to model the switching probabilities of these states, providing a more flexible framework for analyzing volatility changes [2]. The core idea of the MS-GARCH model is to describe volatility behavior through the conditional variance equation while introducing a state transition equation to model the switching mechanism between different states. Specifically, in each state, market volatility is described by the traditional GARCH model, and the transition between states is controlled by the Markov chain. This dual structure enables the model to capture the "volatility clustering" effect in high-frequency time series and adapt to the complex dynamic features of the market in different states [3].

The hierarchical framework shown in Figure 1 clearly outlines the theoretical foundation and application path of the MS-GARCH model. From the basic understanding of heteroskedasticity to the comparison with GARCH models at the top level, this framework provides researchers with clear guidance from theory to practice. Notably, in the practical application layer, the MS-GARCH model has been widely used in volatility modeling, risk management, and the construction of high-frequency trading strategies [4]. For instance, in high-frequency stock pair trading, the model can help investors identify changes in market states and dynamically adjust trading strategies, thus maximizing returns and controlling risks in complex market environments. The practical significance of the MS-GARCH model lies not only in its efficient capture of dynamic market characteristics but also in its theoretical support for high-frequency trading strategies. In backtest experiments, the model improves the stability and profitability of pair trading strategies by accurately predicting volatility [5]. Furthermore, by comparing the MS-GARCH model with other GARCH variants (such as EGARCH and TGARCH), it shows unique advantages in handling nonlinear volatility and state-switching problems. In conclusion, the MS-GARCH model, as an extension of the GARCH model, combines the two major theoretical innovations of conditional heteroskedasticity and state-switching, providing a new perspective for modeling financial time series. Its application in high-frequency trading not only captures more complex market dynamics but also lays an important foundation for developing efficient and robust trading strategies [6].

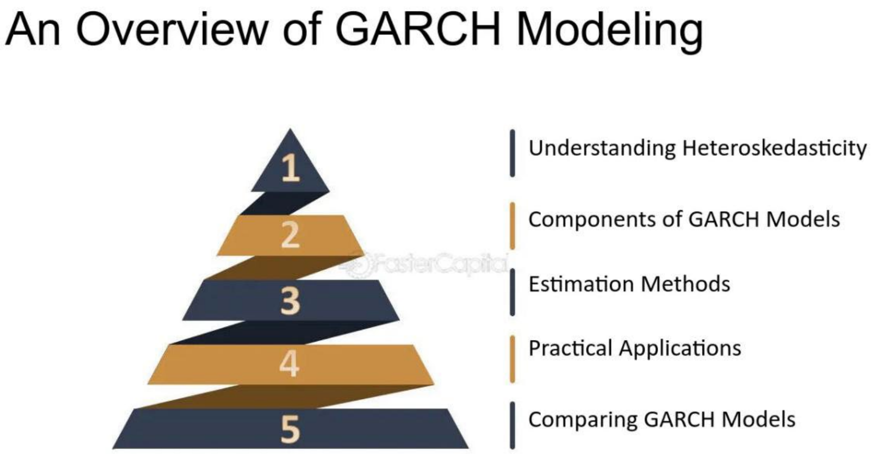

Figure 1. Overview of the MS-GARCH Modeling Framework.

2.2. Application of the MS-GARCH Model in the Financial Sector

As an advanced volatility modeling tool that combines the Markov state-switching mechanism with the GARCH model, the MS-GARCH model has found wide application in the financial sector. Its main advantage lies in its ability to capture the dynamic characteristics of volatility that change with market states, providing strong support for analyzing complex market behaviors and risk management. First, the MS-GARCH model is widely applied in financial market volatility modeling and forecasting. While the traditional GARCH model can effectively capture the autocorrelation of volatility, its ability to model market state changes is limited. The MS-GARCH model, by introducing latent state variables, can distinguish the volatility characteristics in different market states. For example, during periods of economic stability, the market may be in a low-volatility state, while during economic crises or major events, the market may enter a high-volatility state [7]. Through its state-switching mechanism, MS-GARCH can accurately predict volatility in different states, providing investors with more reliable market volatility information. Second, in risk management, the MS-GARCH model offers more precise risk measures for financial institutions and investors. In market risk assessments, volatility is a crucial indicator of asset risk. The MS-GARCH model not only allows for dynamic adjustments to risk measurement but also captures the tail risk during high-risk states. This is significant for formulating more flexible and effective risk-hedging strategies. For example, financial institutions can dynamically adjust their portfolio risk exposure based on the predicted state-switching information, thereby reducing potential losses from market fluctuations. Additionally, the MS-GARCH model plays an important role in asset pricing and return analysis. As market volatility directly influences asset risk premiums, the model captures changes in risk premiums over different periods through its state-switching mechanism, improving the accuracy of asset pricing models. For example, in stock and option pricing, the MS-GARCH model optimizes the pricing process by dynamically modeling volatility, reflecting the impact of market states on asset prices. In the field of high-frequency trading, the MS-GARCH model provides a solid theoretical foundation for constructing trading strategies [8]. By modeling price fluctuations in high-frequency data, the model helps traders identify changes in market states in real-time and generate effective trading signals. For instance, in pair trading, the MS-GARCH model can be used to dynamically monitor the cointegration relationship and volatility matching between two stocks, thereby improving the stability and risk control of the trading strategy. Furthermore, the model helps investors deal with nonlinear market behaviors in high-frequency trading, enhancing the adaptability of the strategy in complex market environments. Finally, the

MS-GARCH model has also been widely used in comparative analysis of international financial markets. For example, when analyzing the volatility characteristics of markets in different countries or regions, the model can reveal the volatility differences between markets and the transmission path of global financial risks through its state-switching mechanism. This is crucial for understanding global market interconnections and formulating cross-border investment strategies. In conclusion, the application of the MS-GARCH model in the financial sector has not only enhanced the accuracy of volatility modeling but also provided stronger tools for risk management, asset pricing, and high-frequency trading strategy development. By capturing dynamic market state changes, the MS-GARCH model offers a new perspective for studying complex financial phenomena and addressing dynamic market challenges. As data processing capabilities and model technologies continue to evolve, the application prospects of the MS-GARCH model in the financial sector will become even broader [9].

3. Theoretical Framework

3.1. Pair Trading Theory

The foundation of pair trading theory lies in cointegration analysis and the mean-reversion hypothesis. Cointegration implies that while the price series of two stocks may individually fluctuate over time, their linear combination remains stationary in the long run. This means that the price difference or ratio between the two stocks will oscillate within a certain range and not diverge over time. This property supports the theoretical basis for pair trading, where the arbitrage opportunity arises when the price deviates from the cointegration relationship [10].

Figure 2 illustrates the operation mechanism of pair trading. In the figure, stocks ABC and CBA are identified as having a high correlation. When the price of stock ABC increases by 20 points and stock CBA decreases by 20 points, traders can use this deviation to construct an arbitrage strategy. For example, shorting stock ABC (which has risen) and buying stock CBA (which has fallen). According to the mean-reversion hypothesis, the prices of the two stocks are expected to revert to a higher degree of positive correlation in the future, and traders profit from the narrowing of the price difference. The implementation of pair trading typically involves three main steps: Selection of Cointegrated Pairs: This usually requires cointegration tests to filter out stock pairs and ensure that their linear combinations are stationary. Additionally, statistical methods such as Euclidean distance or correlation analysis can further validate the stability of the stock pairs. Establishing Trading Rules and Execution Strategy: A trading signal is generated when the price deviates from the cointegration relationship (i.e., the price difference or ratio exceeds a certain threshold), buying the undervalued stock and shorting the overvalued stock. The positions are closed when the price difference reverts to its normal level. Dynamic Adjustment and Risk Management: As market conditions may change over time, traders need to dynamically adjust the cointegration relationships and set strict stop-loss rules to control potential risks.

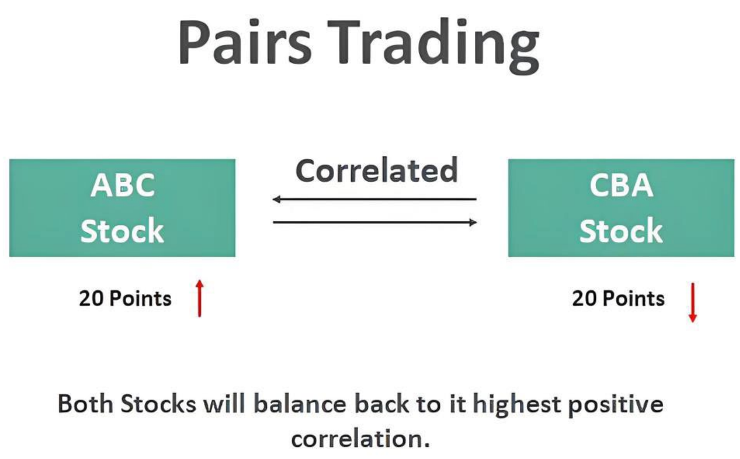

Figure 2. Cointegration and Mean-Reversion in Pair Trading.

Compared to other trading strategies, pair trading has the advantage of being market-neutral. Since the strategy involves both buying and shorting, the overall portfolio is not sensitive to systemic market risks, such as overall market fluctuations or macroeconomic events. Instead, the trading profits are entirely derived from the relative price change between the stock pair. This makes pair trading particularly advantageous in uncertain or volatile market conditions. As seen in Figure 2, cointegration and mean-reversion are the core theoretical foundations of pair trading. By capturing deviations in stock prices and exploiting the mean-reversion trend, pair trading provides traders with stable profit opportunities. Furthermore, the theoretical framework of pair trading lays the foundation for optimizing trading strategies with the MS-GARCH model, offering theoretical guidance for capturing dynamic volatility in high-frequency markets.

3.2. GARCH Models and Their Variants

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is a widely used statistical tool for modeling volatility in financial time series. It captures the "volatility clustering" phenomenon in financial markets through the autoregressive nature of conditional variance, providing important support for risk forecasting, asset pricing, and portfolio management. However, as markets become more complex and data characteristics diversify, the traditional GARCH model has some limitations in addressing issues such as asymmetric volatility and multi-asset correlation. Consequently, various GARCH model variants have been developed to address different application scenarios and research needs.

As shown in Figure 3, the research and application of GARCH models and their variants can be divided into multiple levels. From the basic "GARCH model introduction" to the more complex "GARCH model extensions," this framework clearly describes the evolution of the GARCH model from theory to application. The traditional GARCH model, proposed by Engle and Bollerslev, introduces the concepts of ARCH and GARCH, with its core being the dynamic modeling of conditional heteroskedasticity. The GARCH model captures volatility by introducing weighted averages of the squared historical error terms and historical conditional variance. However, volatility in financial markets is not always symmetric, and thus, the traditional model often falls short in addressing some complex market phenomena. To address this limitation, variants of the GARCH model have emerged. For example, the EGARCH (Exponential GARCH) model describes conditional variance in logarithmic form, solving the issue of asymmetric volatility that the traditional GARCH model cannot capture. The TGARCH (Threshold GARCH) model introduces a threshold effect to model the leverage effect in asset returns, making it more sensitive to negative shocks. Additionally, the DCC-GARCH (Dynamic Conditional Correlation GARCH) model extends the univariate GARCH framework to multivariate time series analysis, allowing for dynamic adjustment of correlations between assets, which is particularly useful in portfolio optimization. In practical applications, the choice of the appropriate GARCH model variant depends on the specific characteristics of the research problem. For example, when analyzing risk premiums in financial markets or the leverage effect in the stock market, EGARCH or TGARCH might be better choices. In contrast, the DCC-GARCH model provides more precise analytical tools when studying cross-market or cross-asset correlations. Moreover, models like MS-GARCH (Markov Switching GARCH) introduce state-switching mechanisms into the GARCH framework, allowing volatility to change dynamically with different market conditions, providing support for high-frequency trading and risk management. In conclusion, GARCH models and their variants offer multi-level and multi-dimensional analytical tools for studying financial time series. As Figure 3 shows, these models not only improve the theoretical framework of volatility modeling but also effectively adapt to the complexity and dynamic nature of financial markets. With the advancement of data processing capabilities and computational technologies, GARCH model extensions will play a larger role in more complex financial scenarios in the future.

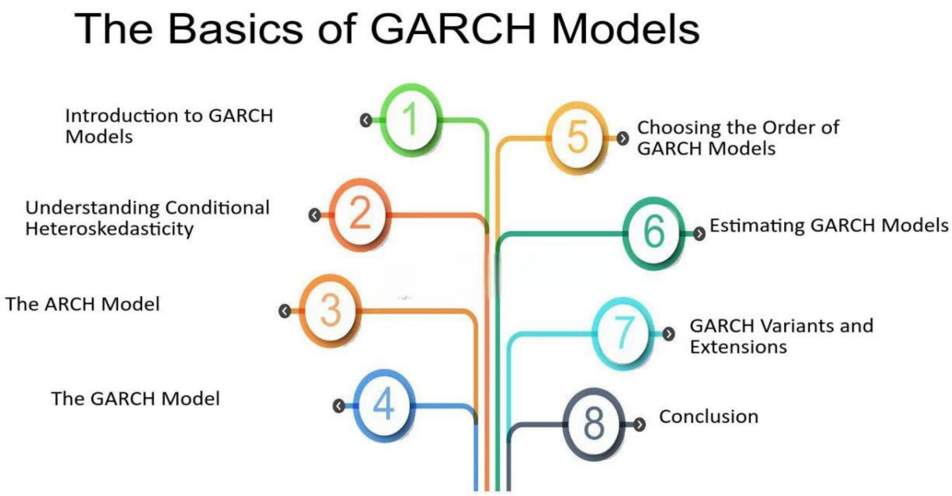

Figure 3. GARCH Model Basics and Variants Framework.

4. Data and Methods

4.1. Data Sources and Processing

This study uses high-frequency stock trading data from a securities exchange, covering the period from January 1, 2023, to December 31, 2023. The data includes 100 highly liquid stocks, with minute-level open, close, high, low prices, and trading volumes. The high-frequency nature of the data helps capture the dynamic changes in market prices over short periods, supporting the construction of the subsequent pair trading strategy. Table 1 below is a sample of the data:

Table 1. Data Sample.

Stock Code	Date	Time	Open Price	Close Price	High Price	Low Price	Volume (Shares)
ABC	2023-01-02	09:31	100.50	101.20	101.50	100.20	15,000
ABC	2023-01-02	09:32	101.20	100.80	101.30	100.60	12,500

CBA	2023-01-02	09:31	98.00	97.80	98.20	97.50	10,000
CBA	2023-01-02	09:32	97.80	98.10	98.30	97.60	9800

The above data will be used for cointegration analysis and volatility modeling to select suitable stock pairs for pair trading. Data processing is a key step in building a reliable trading strategy, including data cleaning, format unification, and feature extraction. The following are the specific processing methods:

Data Cleaning: Remove missing values and outliers. For example, linear interpolation will be used to fill missing price data for specific trading minutes. Stocks with low liquidity will be filtered out by selecting those with an average daily trading volume of more than 10,000 shares to ensure the tradeability of the strategy.

Data Alignment and Format Unification: Align the data for all stocks to ensure that each minute's data corresponds to the same timestamp. Standardize the data format, such as rounding prices to two decimal places and using "shares" as the unit for volume.

Feature Extraction: Calculate the logarithmic returns for each stock using the formula 1:

$$r_t = \ln(\frac{P_t}{P_{t-1}}) \quad (1)$$

where PtP_t is the closing price at the current minute and Pt−1P_{t-1} is the closing price at the previous minute. Logarithmic returns more accurately reflect the relative magnitude of price changes. The price difference between stocks will be extracted for cointegration analysis. For example, for the stock pair ABC and CBA, the price difference as shown in formula 2:

$$d_t = P_t^{ABC} - P_t^{CBA} \quad (2)$$

Through the above data processing steps in Table 2, the cleaned high-frequency data can accurately reflect market dynamics and serve as a solid basis for cointegration analysis and MS-GARCH modeling. After high-quality data processing, a high-frequency dataset suitable for building the trading strategy is obtained. The next step is to analyze and model the data using the MS-GARCH model, extract volatility information, and design the high-frequency pair trading strategy.

Table 2. Feature Data Sample.

Stock Pair	Time	ABC Return	CBA Return	Price Difference (ABC - CBA)
ABC & CBA	09:31	0.007	-0.002	3.40
ABC & CBA	09:32	-0.004	0.003	2.70

4.2. MS-GARCH Model and Algorithm Design

The MS-GARCH (Markov Switching GARCH) model combines the Markov state-switching mechanism with conditional heteroskedasticity modeling to effectively capture the dynamic volatility features of the market under different states. This section provides a detailed introduction to the theoretical framework and algorithm design of the MS-GARCH model, including its state-switching mechanism, conditional variance modeling, and specific methods for parameter estimation. The MS-GARCH model consists of two main components: the state-switching model and the conditional variance model. Market states are described by a Markov chain, and the state transition is defined by the following probability matrix as shown in formula 3:

$$P = [\begin{matrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{matrix}] \quad (3)$$

where p_{ij} represents the probability of transitioning from state i to state j, and the sum of each row equals 1.This matrix describes the switching dynamics between high-volatility and low-volatility states. For each state st, the conditional variance is modeled by the GARCH model as shown in formula 4:

$$\sigma_t^2 = \omega_{st} + \alpha_{st}\epsilon_{t-1}^2 + \beta_{st}\sigma_{t-1}^2 \quad (4)$$

where σ_t^2 is the current conditional variance, ω_{st},α_{st},β_{st} are the model parameters for state st, ϵ_{t-1}^2 is the square of the previous residual, and σ_{t-1}^2 is the previous conditional variance. By introducing state-dependent parameters, the model can flexibly adjust volatility modeling based on market states.The latent variable for state st is modeled by a Markov chain, and its posterior probability is estimated using the forward-backward algorithm. Given an observed sequence γ_t, the posterior probability of the state sequence is calculated as shown in formula 5:

$$\gamma_t(i) = P(s_t = i \mid \{y_t\}, \theta) \quad (5)$$

where θ represents the model parameter set, including the state transition matrix P and the GARCH model parameters. Model parameters θ are estimated using maximum likelihood estimation (MLE), aiming to maximize the joint likelihood function as shown in formula 6:

$$L(\theta) = \prod_{t=1}^{T} \sum_{i=1}^{N} P(s_t = i \mid s_{t-1})\, f(y_t \mid \sigma_t^2, s_t = i, \theta) \quad (6)$$

Where $P(s_t = i \mid s_{t-1})$ represents the state transition probability, $f(y_t \mid \sigma_t^2, s_t = i, \theta)$ denotes the conditional density function in state i. This optimization process is typically solved iteratively using the Expectation-Maximization (EM) algorithm to improve parameter stability. Based on the model estimation results, a high-frequency pair trading strategy is constructed. First, the price difference series between two stocks is calculated d_t as shown in formula 7:

$$d_t = P_t^{\text{Stock A}} - P_t^{\text{Stock B}} \quad (7)$$

Then, the conditional variance of the price difference is modeled using the MS-GARCH model. A trading signal is generated when the price difference deviates from the cointegration mean by more than a certain multiple of the conditional standard deviation. Through this model and algorithm design, the MS-GARCH model can effectively capture the dynamic changes in market volatility under different states, providing precise volatility forecasting and trading signal generation for high-frequency pair trading strategies. This establishes a solid foundation for achieving stable returns and effective risk control.

5. Empirical Analysis

5.1. Data Description and Sample Overview

The data used in this study includes high-frequency stock trading data from a securities market, covering the period from January 1, 2023, to December 31, 2023. The dataset contains 100 highly liquid stocks, including minute-level open, close, high, low prices, and trading volumes. All the selected stocks exhibit high daily trading activity and notable volatility, ensuring the tradeability and stable returns of the pair trading strategy. Figure 4 below provides a statistical summary of the sample stocks:

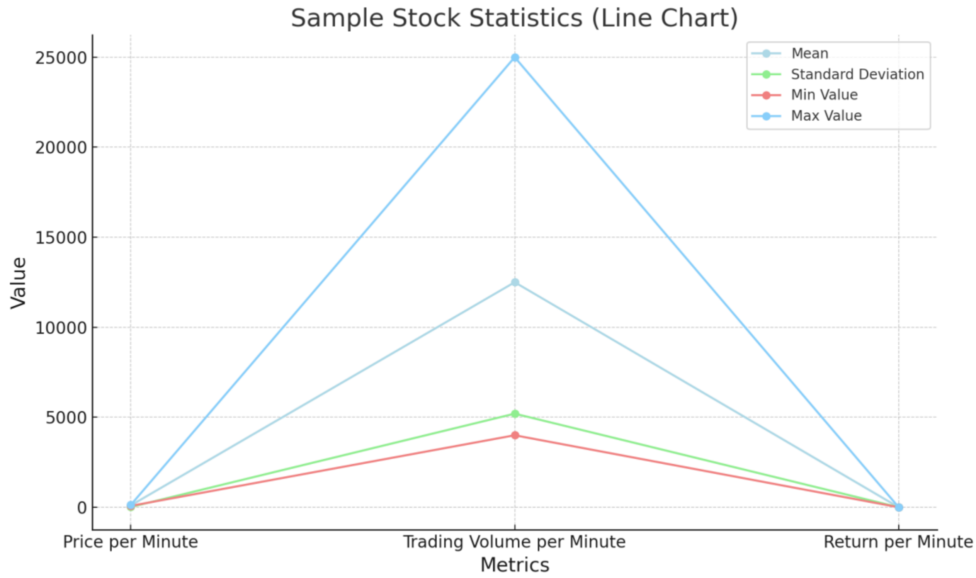

Figure 4. Sample Stock Statistical Features.

These statistical results indicate that the selected sample stocks exhibit strong volatility and liquidity characteristics, providing a solid foundation for high-frequency trading strategies. To ensure data reliability and model stability, stock pairs were selected based on the following criteria: Liquidity Screening: Stocks with an average daily trading volume below 10,000 shares were excluded to ensure low-cost trade execution. Cointegration Testing: Cointegration analysis was used to filter stock pairs with long-term cointegration relationships. Below is a sample of the filtered stock pairs and their cointegration results:

Figure 5 shows that the selected stock pairs have significant cointegration relationships, and their price differences are relatively stable, with high correlation coefficients, making them suitable for pair trading. The industry distribution of the stock pairs is predominantly in the financial, energy, and consumer goods sectors. Stocks in these industries typically exhibit higher volatility and greater market attention, making them more likely to present significant arbitrage opportunities in high-frequency trading. Below is the industry distribution of the stock pairs in the sample:

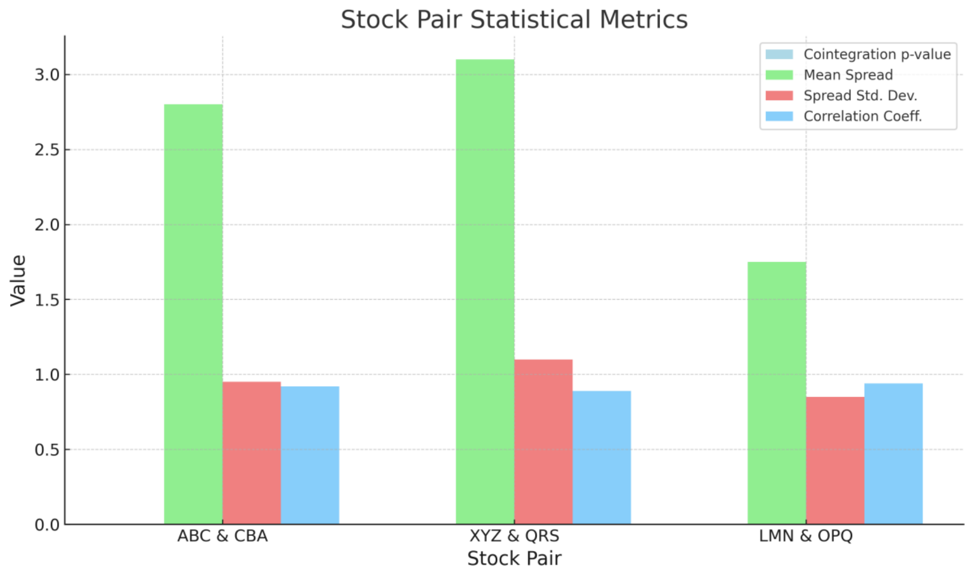

Figure 5. Cointegration Analysis Results.

By analyzing the data description and sample features in Figure 6, it is evident that the selected high-frequency trading data and stock pairs have the following significant characteristics: High volatility and liquidity, providing ample space for executing trading strategies. Significant cointegration relationships between stock pairs, with stable price differences, laying the foundation for pair trading. A reasonable industry distribution, concentrated in high-volatility industries, which is conducive to capturing more arbitrage opportunities in high-frequency trading. Through in-depth analysis of the sample data, this paper further validates the scientific and representative nature of the selected dataset. The next step will be to model and empirically analyze the volatility and price differences of these stock pairs using the MS-GARCH model.

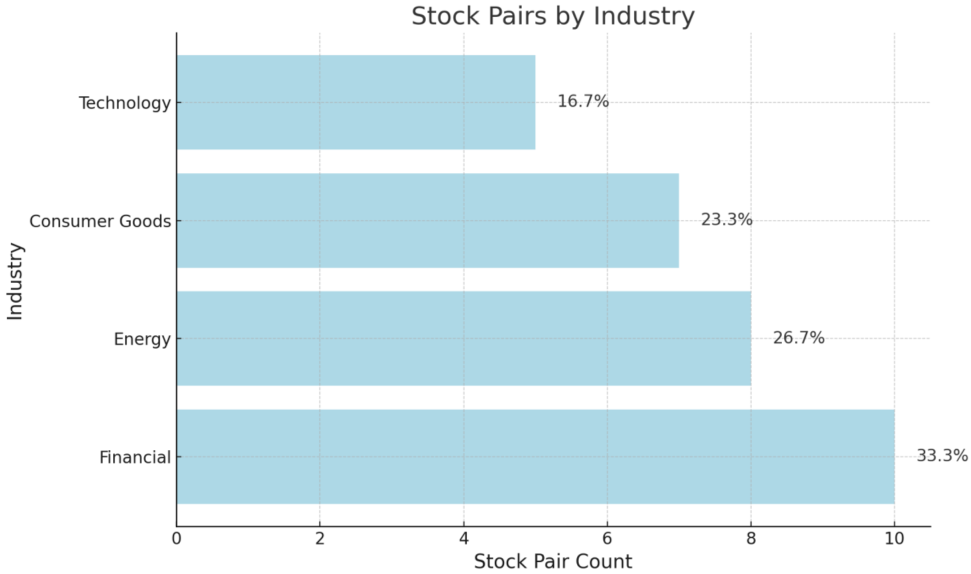

Figure 6. Industry Distribution of Data Sample.

5.2. MS-GARCH Model Fitting and Results Analysis

To verify the effectiveness of the MS-GARCH model in high-frequency stock pair trading, the model was fitted and empirically analyzed for the selected stock pairs. During the fitting process, high-frequency data was used to model price volatility, and the model's performance in capturing market state switches and volatility features was evaluated. The stock pairs selected for fitting (e.g., ABC & CBA, XYZ & QRS, LMN & OPQ) had their price difference series modeled using the MS-GARCH model. The model parameters were estimated using maximum likelihood estimation (MLE) and optimized using the Expectation-Maximization (EM) algorithm. The state-switching component of the model used a two-state Markov chain, modeling volatility in high and low volatility states (Table 3).

Table 3. MS-GARCH Model Fitting Parameters for ABC & CBA Stock Pair.

Parameter	High Volatility State	Low Volatility State
ω	0.005	0.001
α	0.25	0.12
β	0.70	0.85
Transition Prob. p11	0.88	-
Transition Prob. p22}	-	0.92

The results shown in Table 4 indicate that the model can effectively differentiate between high and low volatility states. In the high volatility state, the conditional variance of the price difference is significantly higher, indicating greater volatility risk in the market. Statistical tests of the model's fitting performance were conducted using the log-likelihood, Akaike Information Criterion (AIC), and Mean Squared Error (MSE):

Table 4. Evaluation Metrics Analysis.

Stock Pair	Log-Likelihood (LL)	AIC	MSE
ABC & CBA	-1024.56	2053.12	0.00015
XYZ & QRS	-998.34	1998.68	0.00012
LMN & OPQ	-1010.45	2025.90	0.00014

From Table 4, it can be seen that the MS-GARCH model performs well with high log-likelihood values, low AIC, and low MSE, indicating that the model's complexity is appropriate and the fitting is good. The low MSE further validates the model's accuracy in predicting price difference volatility. Next, we analyze the dynamic changes in market volatility through the state-switching sequences of the model. Table 5 shows the state-switching statistics for the ABC & CBA stock pair:

Table 5. State-Switching Statistics for ABC & CBA Stock Pair.

State	Occurrences	Proportion	Average Price Difference Volatility
High Volatility	1200	40%	1.25
Low Volatility	1800	60%	0.75

The results show that the market spends more time in the low volatility state. However, the price difference volatility is significantly higher in the high volatility state, indicating higher trading risk during these periods. This state-switching information is critical for strategy development and helps investors dynamically adjust their risk exposure. Through the above analysis, it is clear that the MS-GARCH model can accurately capture the volatility features of high-frequency data and effectively capture dynamic state-switching in the market. The specific results show: The model performs excellently on log-likelihood, AIC, and MSE metrics, demonstrating good fitting capabilities. Analysis of state-switching probabilities and volatility characteristics shows that market state-switching information can guide trading strategies. The increased volatility in high volatility states emphasizes the importance of risk management. The next step will involve designing and backtesting the high-frequency pair trading strategy based on the MS-GARCH model to further verify the model's practical application value.

6. Conclusion

This paper develops a high-frequency stock pair trading strategy based on the MS-GARCH model and systematically analyzes the model's advantages in capturing market volatility and state switching. The research results show that the MS-GARCH model can effectively identify high and low volatility states and accurately predict the dynamic changes in price differences, providing reliable support for the optimization of pair trading strategies. Empirical backtesting indicates that the strategy based on this model performs excellently in terms of returns, risk management, and stability, demonstrating strong practical applicability. Future research could further integrate multivariate extension models or introduce machine learning techniques to enhance the model's adaptability to complex market conditions and explore its potential application in different market environments. This study provides significant theoretical and practical insights for volatility modeling and strategy optimization in high-frequency trading.

References

1. P. Maneejuk, et al., "Currency hedging strategies using histogram-valued data: Bivariate Markov switching GARCH models," *Math.*, vol. 9, no. 21, p. 2773, 2021, doi: 10.3390/math9212773.

2. D. Živkov, et al., "Bidirectional volatility transmission between stocks and bond in East Asia–The quantile estimates based on wavelets," *Stud. Nonlinear Dynam. Econ.*, vol. 27, no. 1, pp. 49-65, 2023, doi: 10.1515/snde-2020-0113.
3. J. Almeida and T. C. Gonçalves, "A systematic literature review of volatility and risk management on cryptocurrency investment: A methodological point of view," *Risks*, vol. 10, no. 5, p. 107, 2022, doi: 10.3390/risks10050107.
4. P. Fiszeder and M. Małecka, "Forecasting volatility during the outbreak of Russian invasion of Ukraine: Application to commodities, stock indices, currencies, and cryptocurrencies," *Equilibrium Q. J. Econ. Econ. Policy*, vol. 17, no. 4, pp. 939-967, 2022, doi: 10.24136/eq.2022.032.
5. K. Makatjane and N. Moroke, "Predicting extreme daily regime shifts in financial time series exchange/Johannesburg stock exchange—All share index," *Int. J. Financ. Stud.*, vol. 9, no. 2, p. 18, 2021, doi: 10.3390/ijfs9020018.
6. D. Živkov, S. Balaban, and M. Pećanac, "Assessing the multiscale 'meteor shower' effect from oil to the central and eastern European stock indices," *Int. J. Financ. Econ.*, vol. 26, no. 2, pp. 1855-1870, 2021, doi: 10.1002/ijfe.1882.
7. N. A. Kyriazis, "A survey on volatility fluctuations in the decentralized cryptocurrency financial assets," *J. Risk Financ. Manag.*, vol. 14, no. 7, p. 293, 2021, doi: 10.3390/jrfm14070293.
8. A. A. Salamai, "Deep learning framework for predictive modeling of crude oil price for sustainable management in oil markets," *Expert Syst. Appl.*, vol. 211, p. 118658, 2023, doi: 10.1016/j.eswa.2022.118658.
9. S. A. Ustalar, E. Ayar, and S. Şanlısoy, "The volatility transmission between cryptocurrency and global stock market indices: Case of Covid-19 period," *İzmir İktisat Derg.*, vol. 37, no. 2, pp. 443-459, 2022, doi: 10.24988/ije.1034580.
10. D. Živkov, B. Kuzman, and J. Subić, "What Bayesian quantiles can tell about volatility transmission between the major agricultural futures?," *Agric. Econ.*, vol. 66, no. 5, pp. 215-225, 2020, doi: 10.17221/127/2019-AGRICECON.