Simen Owen Academic Proceedings Series

4th International Conference on Electronics, Engineering, Computer Science and Applied Development (EESD 2026)

Article

Crisis and Hope in Quantum Field Theory through Dimensional Analysis and Geometric Resummation

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Abstract: Quantum field theory organizes interactions into calculable contributions, but the usefulness of a perturbative expansion depends on its physical scales and coefficients. This article examines the contrast between quantum electrodynamics and gravity through dimensional analysis, then considers whether reorganizing a divergent expansion can provide insight into unfamiliar gravitational behavior. The electromagnetic fine-structure constant illustrates the value of a dimensionless coupling without guaranteeing convergence. For gravity, the dimensionless parameter associated with a characteristic microscopic energy grows as the square of its ratio to the Planck energy, indicating where a low-energy expansion may lose control. A geometric-series model makes the distinction between convergence and analytic continuation explicit. Its continued expression changes sign across a pole, but that mathematical property does not establish gravitational repulsion. Connecting such a proposal to cosmology requires a physical observable and consistent equations for spacetime evolution. Established inflationary mechanisms provide examples of how specified dynamics can produce accelerated expansion. The article argues that the promise of perturbative breakdown lies in identifying the additional structure needed for a controlled description, while retaining dimensional analysis and mathematical reformulation as useful starting points for theoretical exploration.

Keywords: quantum field theory; dimensional analysis; quantum gravity; perturbation theory; resummation; cosmic inflation

Received: 09 September 2026
Revised: 18 October 2026
Accepted: 31 October 2026
Published: 07 November 2026

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1. Introduction

Quantum field theory (QFT) describes interactions through quantum fields and provides a systematic connection between theoretical amplitudes and observable probabilities. Feynman diagrams organize terms in a perturbative calculation: tree diagrams give the lowest-order contributions where these exist, while loop diagrams add corrections. Dyson's formulation of the QED scattering matrix established how this organization works together with renormalization [1]. Its success is illustrated by Schwinger's leading correction to the electron's magnetic moment [2]. Such achievements invite a question that becomes especially important for gravity: what happens when the corrections cease to be small?

For a QED process with a nonzero two-vertex tree contribution, the amplitude can be written schematically as

$$\mathcal{M}_{QED} = \alpha(\mu)C_0 + \alpha^2(\mu)C_1 + \alpha^3(\mu)C_2 + \cdots \quad (1)$$

Here α is the renormalized electromagnetic coupling, μ is a renormalization scale, and the coefficients contain the dependence on external momenta, masses, and other kinematic quantities. Normalization and numerical loop factors are absorbed into the coefficients. The equation organizes powers of a coupling; it does not assert that the

coefficients are equal or that the expansion converges. A scattering amplitude is generally complex and enters probabilities through its modulus squared, rather than directly describing the direction of a force.

The distinction between a useful approximation and a convergent infinite series is fundamental. Dyson's argument for divergent QED perturbation theory already showed why accurate low-order predictions should not be mistaken for a demonstration of convergence [3]. Gravity introduces a further issue because its coupling carries dimensions that make quantum corrections increasingly sensitive to energy. Nevertheless, general relativity remains predictive as a quantum effective field theory at sufficiently low energies [4-5].

This article follows a progression from electromagnetic dimensional analysis to gravitational power counting, geometric resummation, and cosmological interpretation. Its central question is whether a change in mathematical representation can reveal physics beyond a failing expansion. The analysis uses established results and an explicitly defined toy model. It develops the possibility of altered gravitational behavior as a research question whose answer requires more information than the sign of a continued series.

2. Theoretical Framework and Method

2.1. Dimensionless Electromagnetic Coupling and Perturbative Control

The dimensional comparison begins with the electrostatic potential energy of two opposite elementary charges separated by a distance r. In SI units,

$$V_C(r) = -\frac{e^2}{4\pi\varepsilon_0 r} \tag{2}$$

Multiplying the potential by distance gives a quantity with dimensions of energy times length. The combination ħc has the same dimensions, allowing a dimensionless comparison:

$$\alpha(0) = \frac{|V_C(r)r|}{\hbar c} = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137} \tag{3}$$

The absolute value removes the sign associated with the chosen charge pair. The positive constant α measures the electromagnetic interaction strength, and its low-energy value is approximately 1/137. The exercise reveals which combination of the charge and fundamental constants can control an expansion. It does not remove the dependence of an amplitude on momenta or masses. In units where ħ and c equal one, the same physical comparison remains present even though the conversion factors disappear.

Dimensionless quantities can still depend on physical scales. Vacuum polarization changes the effective electromagnetic interaction with momentum transfer, a phenomenon rooted in the renormalization analysis of Gell-Mann and Low [6]. The Particle Data Group distinguishes the low-energy value from the effective coupling at higher scales [7]. Consequently, replacing a dimensionful charge by α clarifies the bookkeeping but cannot establish complete separation between coupling and kinematics. Renormalized coefficients and the running coupling depend on the chosen description; a physical observable must be independent of that arbitrary choice when all required contributions are included.

Tree-level dominance therefore requires an additional assessment. Successive terms must actually be smaller in the regime under consideration. Large logarithms of separated scales, exceptional kinematics, or an absent leading contribution can change the counting. A process with no allowed tree diagram begins at a loop order, so the general success of tree approximations elsewhere says little about its leading term. The relevant comparison is between the sizes of complete contributions to a specified observable.

This can be seen without evaluating an entire series. Provided the leading coefficient is nonzero, the relative first correction is the coupling multiplied by the ratio of the first two coefficients. A small coupling only suppresses that correction when the coefficient ratio does not compensate for it. Even then, the comparison concerns the first correction rather than every subsequent order. Renormalization-scale variation and explicit higher-order calculations can help assess uncertainty, but neither is a substitute for specifying the

process. This is the practical meaning of perturbative control: a reasoned estimate of what a chosen truncation omits.

Schwinger's result, that the leading electron magnetic anomaly equals α divided by 2π, illustrates a calculable small correction [2]. Its smallness follows from an evaluated coefficient together with a small coupling, rather than dimensional analysis alone. Conversely, an asymptotic series can give excellent predictions when truncated appropriately even if adding terms indefinitely eventually worsens the approximation. Dyson's argument concerns this large-order behavior [3]. The practical lesson is to identify a useful domain and estimate omitted contributions, rather than require an infinite series to converge before accepting any finite-order prediction.

2.2. Gravitational Coupling and the Planck Scale

The Newtonian potential provides a parallel dimensional illustration. For two slowly moving masses, the potential energy is negative and proportional to their mass product. Writing their rest energies as E_1 and E_2 gives

$$V_N(r) = -\frac{Gm_1m_2}{r}, \quad \frac{|V_N r|}{\hbar c} = \frac{Gm_1m_2}{\hbar c} = \frac{GE_1E_2}{\hbar c^5} \qquad (4)$$

For equal rest energies the numerator contains E squared. This resemblance motivates the definition of a dimensionless gravitational strength. When applying quantum power counting to relativistic scattering, however, E must denote a characteristic invariant microscopic energy or momentum scale, rather than an arbitrary system's total rest energy. With this interpretation,

$$E_P \equiv \sqrt{\frac{\hbar c^5}{G}}, \quad \alpha_G(E) \equiv \frac{GE^2}{\hbar c^5} = \left(\frac{E}{E_P}\right)^2 \qquad (5)$$

The unreduced Planck energy is approximately 1.22×10^{19} GeV [8]. Some literature uses a reduced Planck scale differing by a numerical factor involving 8π; the convention in Eq. (5) is retained throughout. Well below this scale, the energy ratio supplies a small parameter. Near it, that particular suppression is no longer available. Figure 1a illustrates this dimensional growth without assigning an exact energy to a physical transition.

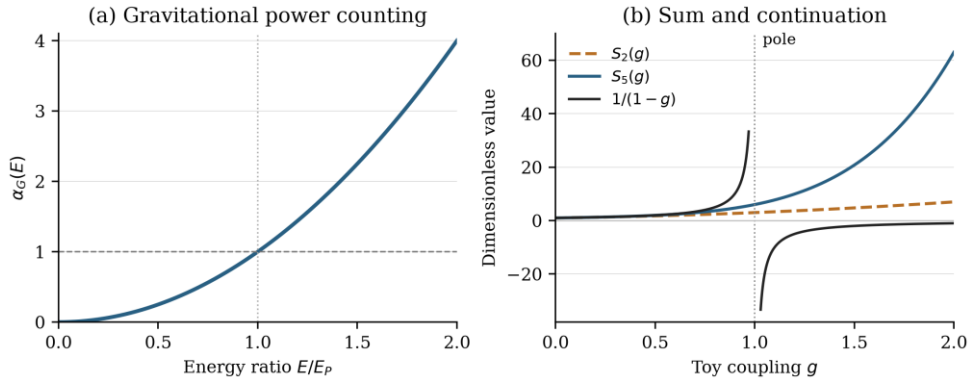


Figure 1. Mathematical Illustrations of the Dimensional Argument and Geometric Model. (A) α_G Grows Quadratically with the Energy Ratio; the Reference Line at Unity Indicates Loss of a Small Parameter, Not a Demonstrated Transition. (B) Finite Geometric Sums and the Continued Rational Function Agree Progressively Below G = 1 but Behave Differently Above It. The Gap in the Rational Curve Marks Its Pole. Curves Are Calculated from the Displayed Equations and Are Not Experimental Data.

For orientation, an energy equal to one thousandth of the Planck energy gives a dimensional parameter of one millionth. At one tenth of the Planck energy, the parameter is one hundredth. These are arithmetic consequences of the quadratic relation, rather than predictions for the numerical size of a particular measured correction. Their value is comparative: raising the energy changes the available suppression rapidly. The calculation does not identify a sudden physical boundary, because it contains no information about the coefficients or about additional thresholds that a more complete theory might introduce.

For a dimensionless two-to-two amplitude at fixed scattering angles with a nonzero Einstein-gravity tree term, the organizing structure is schematically

$$\mathcal{M}_G(E) \sim (\frac{E}{E_P})^2[B_0 + B_1(\frac{E}{E_P})^2 + B_2(\frac{E}{E_P})^4 + \cdots] \quad (6)$$

The coefficients depend on the process and include numerical factors, logarithms, and angular dependence. Contributions from additional effective operators also enter the power counting. Neither Eq. (5) nor Eq. (6) proves that all orders become equal at the Planck energy. They identify the loss of a parametrically small ratio, which is sufficient reason to question an extrapolation. Burgess develops this distinction between estimates from power counting and actual amplitude calculations [5].

The interpretation of E prevents a misleading application to ordinary objects. A planet's rest energy can exceed the Planck energy while its long-distance gravitational behavior remains predominantly classical. Large mass, strong classical curvature, and large quantum corrections are different conditions. Donoghue and Torma show that apparently dangerous mass factors in individual diagrams require attention to cancellations and consistent counting [9]. In a cosmological setting, relevant scales likewise concern local processes and background evolution; the total energy assigned to the entire universe is not a universal quantum expansion parameter.

Gravity's perturbative difficulties also have a precise historical meaning. The one-loop analysis of 't Hooft and Veltman contains important qualifications for pure gravity evaluated on shell [10]. Goroff and Sagnotti established nonrenormalizable divergences at two loops [11]. These results concern the counterterms required by the theory, rather than a universal numerical catastrophe at one specified energy. The effective-field-theory approach accommodates higher-curvature terms and organizes their contributions by the accuracy sought. Only finitely many coefficients contribute at a fixed order, even though the full action contains an infinite sequence of allowed operators [4,12].

Three questions should therefore remain separate: whether an individual loop integral requires renormalization, whether the expansion over orders converges, and whether the physical scale is small enough for the low-energy hierarchy to be useful. Their answers need not coincide. The constructive meaning of the Planck-scale problem is that familiar low-energy information may cease to suffice, leaving room for additional dynamics or a different controlled approximation.

2.3. Geometric Resummation and Analytic Continuation

The desire to reorganize an unreliable expansion motivates a simple experiment. Let g denote the gravitational parameter $\alpha_G(E)$, and suppose, purely as a model, that every successive correction equals the tree amplitude multiplied by another power of g:

$$\mathcal{M}_{toy}(g) = \mathcal{M}_{tree}\sum_{n=0}^{\infty} g^n, \quad 0 \le g < 1 \quad (7)$$

This is a much stronger assumption than dimensional analysis. It fixes infinitely many otherwise unknown coefficients and discards their kinematic dependence. Its usefulness lies in making a mathematical issue transparent. The dimensionless finite partial sum S_N, containing powers from zero through N, is

$$S_N(g) = \sum_{n=0}^{N} g^n = \frac{1-g^{N+1}}{1-g}, \quad g \neq 1 \quad (8)$$

For a real nonnegative g smaller than one, the final power approaches zero as N increases. The ordinary sum then equals the familiar rational expression. For g greater than one, every term remains positive and the partial sums grow without bound. The same rational expression still defines a function away from its pole:

$$\mathcal{M}_{cont}(g) = \frac{\mathcal{M}_{tree}}{1-g}, \quad g \neq 1 \quad (9)$$

The normalized factor $1/(1-g)$ is negative above g = 1. This branch is obtained by analytic continuation, not by taking the limit of the original partial sums. Figure 1b shows the difference directly. At g = 2, for example, a partial sum ending at the fifth power equals 63, while the continued normalized factor equals −1. These values answer different mathematical questions. Changing the interpretation of the latter does not make the former sequence converge.

The finite-sum formula also supplies an exact error estimate inside the convergence region. The difference between the limiting rational value and the partial sum equals the first omitted power divided by one minus g. Relative to the full rational value, the error is simply the first omitted power. Thus, at g = 0.1, retaining the constant, linear, and quadratic terms gives a relative error of one thousandth. This success rests on knowing the coefficients of the model. For an actual field-theory amplitude, replacing unknown coefficients by this pattern removes the very uncertainty that the physical calculation must resolve.

Analytic continuation is nevertheless a legitimate and powerful operation. The challenge is deciding whether a continued function represents the physical quantity of interest. A pole may reflect a meaningful excitation in a well-founded propagator, or it may be an artifact of an approximation. Its interpretation depends on the underlying theory and boundary conditions. A selected denominator cannot supply those conditions by itself. In particular, the pole in Eq. (9) obstructs the claim that the model demonstrates a smooth passage between attractive and repulsive regimes.

Actual Dyson resummation has a more specific structure. Suppressing indices, momentum integrals, and conventional factors, a propagator D and its one-particle-irreducible self-energy Σ obey the schematic identity

$$D = D_0 + D_0 \Sigma D = [{D_0}^{-1} - \Sigma]^{-1} \quad (10)$$

The identity organizes repeated self-energy insertions. If the exact self-energy were known, it would encode far more information than a single dimensionless ratio. In practice, an approximation to the self-energy sums selected classes of diagrams while leaving other contributions untreated. Studies of self-consistent approximations show why preserving relevant symmetry identities requires care [13]. An expression containing infinitely many terms is therefore not automatically a complete nonperturbative solution.

The distinction preserves the motivating intuition while giving it a workable form. A failed expansion can suggest a reorganization, but the reorganized quantity must be derived or otherwise constrained. For gravity, low-energy agreement is one necessary condition; consistency of the additional terms and their physical interpretation are others. The geometric example identifies these requirements clearly because the step from unknown coefficients to a common ratio is visible and testable.

Knowledge of a few low-order terms cannot generally determine a global continuation. The quadratic polynomial consisting of the first three geometric terms and the normalized factor 1/(1 − g) associated with Eq. (9) agree through quadratic order near the origin. Nevertheless, the polynomial stays positive for nonnegative g and has no pole, whereas the rational function has a pole and a negative branch. Higher-order information distinguishes them. This elementary comparison demonstrates why the proposed high-energy sign reversal cannot follow from low-energy agreement alone. A physical resummation must justify its additional information rather than conceal it in a convenient functional form.

3. Physical Implications and Cosmological Applications

3.1. From an Amplitude to an Attractive or Repulsive Interaction

The central proposed physical interpretation is that the sign change in Eq. (9) might indicate repulsion. This requires a careful definition of what changes sign. An overall reversal of a complete scattering amplitude leaves its modulus squared unchanged:

$$|-\mathcal{M}|^2 = |\mathcal{M}|^2 \quad (11)$$

Relative signs between interfering contributions can affect observables, but a global sign alone cannot identify a new force direction. Momentum transfer is a kinematic quantity constrained by the scattering process, and an exchanged graviton is not characterized by a universal switch from negative to positive transferred momentum. Such a switch is not specified by the toy denominator.

An attractive or repulsive conservative interaction can instead be described through a potential in an appropriate limit. Given a potential energy V, the force follows from its

spatial gradient with a minus sign. The Newtonian potential leads to attraction under this prescription. Obtaining a potential from a relativistic amplitude requires a low-energy matching procedure with stated conventions and, when matching a Hamiltonian potential, subtraction of contributions already reproduced by lower-order dynamics. The gravitational scattering analysis of Bjerrum-Bohr, Donoghue, and Holstein makes these definitional issues explicit [14]. A sign in a proposed amplitude is insufficient without this connecting calculation.

The suggestion can thus be formulated as a conditional question: could a controlled extension of the gravitational description yield an effective repulsive contribution in some regime? A candidate model would need to specify the interaction and demonstrate the effect in an observable or consistently defined potential. Recovering the familiar low-energy result would be an essential check. The geometric continuation supplies none of these dynamical results, although it helps expose exactly where they are needed.

A repulsive correction would also differ from a reversal of the total interaction. A small term opposing the leading attraction can reduce its magnitude while leaving the net force attractive. Extrapolating until that correction overtakes the leading term would be unreliable if the same extrapolation invalidates the expansion. A convincing reversal therefore requires control over the complete relevant result in the regime where the reversal is claimed.

This separation also protects the creative part of the argument. The failure of a familiar approximation need not imply that physical phenomena become meaningless. It can indicate that different variables or a different organization are required. However, the choice among possible continuations must be constrained by physics; a preferred qualitative outcome cannot determine that choice in advance.

3.2. Cosmic Expansion and Accelerated Expansion

Cosmology gives the proposed change of behavior a concrete setting, but expansion and repulsion cannot be equated. General relativity already admits expanding universes in which ordinary gravity remains attractive. Expansion describes the evolution of the spatial scale factor a(t), while inflation concerns an interval during which that expansion accelerates. The standard description does not require matter to explode outward into a pre-existing empty space from a nucleus-sized object.

Within Einstein gravity, for a homogeneous and isotropic spacetime, let ε denote total energy density and p pressure, with any cosmological-constant contribution included in them. The acceleration equation is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3p) \tag{12}$$

The factor of c squared reflects the use of energy density rather than mass density [15]. Accelerated expansion requires ε + 3p to be negative within this description. High positive density alone does not meet that condition. Pressureless matter gives deceleration, as does radiation with positive pressure. Thus an early hot state does not by itself explain an inflationary phase through stronger gravitational coupling.

Inflationary proposals supply additional dynamics. Guth's model connects accelerated expansion with a high-energy phase-transition setting [16], while Linde's new-inflation scenario addresses limitations of earlier proposals [17]. Starobinsky's work provides a particularly relevant bridge to the present motivation: quantum corrections to gravitational dynamics can lead to qualitatively different cosmological solutions [18]. That possibility follows from specified equations, not from assigning a sign to a divergent series.

A canonical homogeneous scalar field gives a familiar illustration. In natural units, its energy density and pressure are

$$\varepsilon_\varphi = \frac{\dot{\varphi}^2}{2} + U(\varphi), \quad p_\varphi = \frac{\dot{\varphi}^2}{2} - U(\varphi) \tag{13}$$

Here U(φ) is a potential energy density, distinct from the two-body potential energy V(r). When the potential contribution dominates the kinetic term, the pressure is approximately the negative of the energy density. Under suitable dynamical conditions this can satisfy the acceleration criterion [19]. The example explains how a proposed microscopic description can connect to spacetime evolution. It does not establish that this

mechanism operates at the Planck energy, and many inflationary models are formulated below that scale. The relevant hope is a calculable bridge between quantum physics and cosmology, rather than a universal rule that sufficiently energetic gravity must repel.

3.3. Developing the Hypothesis into a Physical Model

A more complete version of the proposal would start by specifying an action or a controlled set of amplitudes whose low-energy limit reproduces established gravity. Any resummation would then need a stated approximation scheme and an account of which contributions it includes. This would allow the sign-changing term, if one survived, to be distinguished from a denominator introduced solely for mathematical convenience. The microscopic parameter and the cosmological background scales would also have to be related explicitly.

The next question would concern consistency. A candidate interaction must be examined for unitarity, physically acceptable propagation, and instabilities. Its cosmological equations must yield an accelerated solution and explain how that phase ends. These are substantive calculations, because an accelerating solution that cannot evolve into a viable later universe remains incomplete. The history of inflation illustrates why identifying a promising early behavior is only the beginning of constructing a successful scenario [16-17].

Finally, the model would need predictions beyond the existence of expansion. Cosmic microwave background analyses constrain the properties of primordial perturbations, as demonstrated by the Planck Collaboration's inflation study [20]. Predicting the amplitude and scale dependence of these fluctuations would distinguish competing mechanisms that produce a similar homogeneous expansion history. The connection between quantum fluctuations and observed cosmic structure therefore adds information that a background sign argument cannot supply. A proposed quantum-gravity mechanism must eventually meet such observational tests. The present dimensional and geometric analysis does not calculate those predictions; it identifies the logical steps required to turn an exploratory interpretation into a testable theory.

4. Conclusion

Dimensional analysis exposes a useful contrast between electromagnetism and gravity. The electromagnetic coupling is dimensionless and small at low energies, but its scale dependence and perturbative coefficients still matter. Gravity's characteristic coupling grows with the square of the microscopic energy ratio to the Planck scale, indicating where a low-energy hierarchy may become unreliable. This limitation coexists with successful quantum predictions from gravitational effective field theory.

The geometric example explains why reformulation can be informative without settling physical interpretation. Its rational continuation changes sign across a pole, whereas the positive-term series above that pole still diverges. Neither this sign change nor divergence alone establishes repulsion. Cosmological acceleration requires a dynamical account of stress, energy, and geometry, together with consistency and observational tests.

The enduring value of the original question is its willingness to reconsider an approximation when its familiar form fails. That willingness becomes scientifically productive when it leads to sharper definitions, justified continuation procedures, and calculable consequences. The crisis is a boundary of control; the hope is that understanding that boundary can guide a more complete physical description.

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